UNIT-IV					
8	a)	Find the directional derivative of $\varphi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $xlogz - y^2 = -4$ at (-1,2,1)	L4	CO5	5 M
	b)	Prove that $div(r^n\vec{r}) = (n+3)r^n$	L3	CO3	5 M
		OR			
9	a)	Show that $\nabla^2(r^m) = m(m+1)r^{m-2}$	L3	CO3	5 M
	b)	Identify the values of 'a' and 'b' such that the surface ax^2-byz=(a+2)x and $4x^2y+z^3$=4 cut orthogonally at (1,-1,2).	L2	CO3	5 M
		UNIT-V			
10		Verify Green's theorem $\int_C (3x - 8y^2)dx + (4y - 6xy)\,dy$ where'C' is the boundary of the region bounded by the x=0, y=0 and x+y=1.	L4	CO5	10 M
		OR			
11		Calculate $\int_S \bar{F}.\bar{n}ds\ where \bar{F} = 2x^2y\bar{i} - y^2\bar{j} + 4xz^2\bar{k}$ and 'S' is the closed surface of the region in the first octant bounded by the cylinder $y^2 + z^2 = 9$, and the planes x=0, x=2, y=0 and z=0.	L3	CO5	10 M

PVP 23

Code: 23BS1201

I B.Tech - II Semester – Regular Examinations - JULY 2024

DIFFERENTIAL EQUATIONS & VECTOR CALCULUS

(Common for ALL BRANCHES)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Check whether the equation $(sinxcosy + e^{2x})dx + (cosxsiny + tany)dy = 0$ is exact differential equation or not.	L2	CO1
1.b)	Find the integrating factor of $x\frac{dy}{dx} + y = logx$.	L2	CO1
1.c)	Define Auxiliary equation and Wronskian.	L1	CO2
1.d)	Find the Particular integral of $(D^3 + 4D)y = sin2x$.	L1	CO2
1.e)	Form a partial differential equation by eliminating arbitrary constant 'a' from $Z = alog\left(\frac{b(y-1)}{1-x}\right)$.	L3	CO2
1.f)	Form a partial differential equation by eliminating arbitrary function 'φ' from $Z = e^{my}\varphi(x-y)$.	L3	CO2
1.g)	Define directional derivative and gradient of a scalar point function.	L1	CO3

1.h)	Find the $div\vec{F}$ and $curl\vec{F}$ where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.	L3	CO3
1.i)	State the Green's theorem.	L2	CO5
1.j)	If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, Calculate, $\int_c \vec{F}.d\vec{r}$ where 'C' is the curve in the xy-plane $y=2x^2$ from (0,0) to (1,2).	L3	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Solve the differential equation $xy(1+xy^2)\frac{dy}{dx} = 1$.	L3	CO2	5 M
	b)	Find the solution to the differential equation. $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$	L1	CO4	5 M
		OR			
3	a)	A bacterial culture, growing exponentially increases from 200 to 500 grams in the period from 6 am to 9 am. How many grams will be present at noon?	L2	CO4	5 M
	b)	Calculate the general solution of the differential equation $(ylogy)dx + (x - logy)dy = 0$.	L3	CO4	5 M

		UNIT-II			
4	a)	A condenser of capacity C discharged through an inductance 'L'& resistance R in series and the charge 'q' at time 't' satisfies the equation $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$, given that L=0.25 henries, R=250 ohms and $C = 2 \times 10^{-6}\,farads$, and that when t=0, charge q is 0.002 coulombs and the current $\frac{dq}{dt} = 0$, obtain the value of 'q' in terms of 't'.	L4	CO2	5 M
	b)	Solve $(D^2 + D + 1)y = (1 - e^x)^2$.	L3	CO2	5 M
		OR			
5	a)	Calculate the particular integral of $(D^3 + 1)y = \sin(2x + 3)$.	L3	CO2	5 M
	b)	Solve $(D^2 + 4)y = tan2x$.	L3	CO2	5 M
		UNIT-III			
6	a)	Solve the partial differential equation $(D^2 - 2DD' + D'^2)z = e^{x+y}$.	L3	CO2	5 M
	b)	Determine the solution to the equation $xp - yq = y^2 - x^2$.	L3	CO4	5 M
		OR			
7	a)	Solve $\frac{y^2z}{x}p + xzq = y^2$.	L3	CO2	5 M
	b)	Form a partial differential equation by eliminating the arbitrary constants from the differential equation of all spheres whose centres lie on the z-axis.	L2	CO2	5 M

Differential equations & vector Calculus

1 (a) $\frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ —— 2 M

(b) Formula — 1 M

Answer — 1 M

(c) Auxillary equation — 1M

wronskian —— 1M

(d) y_p — 2 M

(e) Derivative w.r.to x — 1M

Derivative w.r.to y — 1M

(f) Derivative w.r.to x, y — 1M

relation — 1M

(g) Directional derivative — 1M

Gradient —— 1M

(h) $\bar{F}$. —— 1 M

$\nabla \cdot \bar{F}, \nabla \times \bar{F}$ —— 1M

(i) Definition —— 2 M

(j) Substitution —— 1M

Answer —— 1M

2 (a) Standard form — 1M

Substitution —— 2 M

Integrating factor — 1 M

Solution —— 1 M

(b) $\frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ —— 3M

Solution —— 2 M

3 (a) Conditions —— 2M

find C, K —— 2 M

Answer — 1 M

3(b) $\frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ — 1M

I.F — 2M

Solution — 2M

4(a) Roots — 2M

$q, \frac{dq}{dt}$ — 2M

A, B — 1M

4(b) y_c — 2M

y_p — 3M

5(a) Formula — 2M

Answer — 3M

(b) y_c, w — 2M

u_1 — 1M

u_2 — 1M

y_p — 1M

6(a) general solution — 2M

particular integral — 3M

(b) C_1 — 2M

C_2 — 2M

$\phi(C_1, C_2) = 0$ — 1M

7(a) C_1 — 2M

C_2 — 3M

(b) equation — 2M

Derivative w.r. to x, y — 2M

Answer — 1M

8(a) ∇f — 2M

$\nabla \phi$ — 2M

$\nabla \phi \cdot \frac{\bar{a}}{|\bar{a}|}$ — 1M

(b) $\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z}$ → 2M

Answer — 3M

9(a) formula — 1M

∇r^m — 2M

$\nabla^2 r^m$ — 2M

(b) $\nabla \phi_1$ — 1M

$\nabla \phi_2$ — 1M

Dot product — 1M

a — 1M

b — 1M

(10) $\frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ — 2M

Along OA — 1M

Along AB — 1M

Along BO — 1M

OA + AB + BO — 1M

Diagram — 1M

$\iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$ — 3M

(11) GDT — 2M

T.F — 2M

Substitution — 2M

Calculation — 4M

23BS1201

Differential equations & vector Calculus

July 2024 (PYP23)

1a. $M = \sin x \cos y + e^{2x}$. $N = \cos x \sin y + \tan y$

$\frac{\partial M}{\partial y} = -\sin x \sin y$ $\quad \frac{\partial N}{\partial x} = -\sin x \sin y$ — 2M

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ equation is exact

1b. $\frac{dy}{dx} + \frac{1}{x} y = \ln x$ is linear equation in y — 1M

$P = \frac{1}{x}$, $I \cdot F = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$ — 1M

1c. Auxillary equation: An equation obtained from the standard form of a linear differential equation by replacing the RHS zero. — 1M

wronskian: wronskian of two function y_1, y_2 is defined as $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ — 1M

1.d. $y_p = \frac{1}{D^3+4D} \sin 2x = x \frac{1}{3D^2+4} \sin 2x = \frac{x \sin 2x}{-8}$ — 2M

1e. $z = a \ln\left[\frac{b(y-1)}{1-x}\right]$

$p = a\left[\frac{\cancel{1-x}}{\cancel{b(y-1)}}\right] \times \frac{\cancel{b(y-1)}}{+(1-x)^{\cancel{2}}} = \frac{a}{1-x}$

$q = a\left[\frac{\cancel{1-x}}{\cancel{b}(y-1)}\right] \times \left[\frac{\cancel{b}}{\cancel{1-x}}\right] = \frac{a}{y-1}$ — 1M

$p/q = \frac{y-1}{1-x}$ is required PDE — 1M

1.f $z = e^{my} \phi(x-y)$

$p = e^{my} \phi'(x-y) \cdot 1$

$q = m e^{my} \phi(x-y) + e^{my} \phi'(x-y)(-1)$ — 1M

$\Rightarrow q = mz - p \Rightarrow p+q = mz$ — 1M

1.g Directional derivative of $\phi(x,y,z)$ in the direction of $\bar{a}$ is $\nabla\phi\cdot\frac{\bar{a}}{|\bar{a}|}$ ——— 1M

Gradient of scalar point function $\phi(x,y,z)$ is

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z} \text{ ——— 1M}$$

1.h

$$\bar{F} = \Sigma i\frac{\partial}{\partial x}(x^3+y^3+z^3-3xyz)$$

$$= (3x^2-3yz)i + (3y^2-3xz)j + (3z^2-3xy)k \text{ ——— 1M}$$

$$\nabla\cdot\bar{F} = \Sigma i\frac{\partial}{\partial x}\cdot\bar{F} = 6x+6y+6z$$

$$\nabla\times\bar{F} = \nabla\times\nabla\phi = \bar{0}$$

——— 1M

1.i Let C be any simple closed curve encloses region R, M, N be functions of x and y whose first order partial derivatives exists and are continuous over R then ——— 1M

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dx\,dy \text{ ——— 1M}$$

1.j

$$\int_C \bar{F}\cdot d\bar{r} = \int_C 3xy\,dx - y^2\,dy$$

$$= \int_0^1 6x^3\,dx - 4x^4\times 4x\,dx \text{ ——— 1M}$$

on $y=2x^2$

$$= \left[\frac{6x^4}{4} - 16\times\frac{x^6}{6}\right]_0^1 = -7/6 \text{ ——— 1M}$$

2(a)

$$\frac{dx}{dy} = xy + x^2y^3 \Rightarrow \frac{dx}{dy} - xy = y^3x^2 \text{ ——— 1M}$$

It is Bernoulli's equation in x

Divide by x^2 and put $x^{-1}=t$

$$-\frac{dt}{dy} - ty = y^3 \Rightarrow \frac{dt}{dy} + ty = -y^3 \text{ is linear in } t \text{ ——— 2M}$$

$$I\cdot F = e^{\int y\,dy} = e^{y^2/2} \text{ ——— 1M}$$

Sol is

$$t\cdot e^{y^2/2} = \int e^{y^2/2}(-y^3)\,dy$$

put $y^2/2 = p$, $y\,dy = dp$

$$= -\int e^p\,p\,dp = -(pe^p - e^p) \text{ ——— 1M}$$

$e^{y^2/2}(\ldots)$ ... required solution.

2(b) $M = ye^{xy}$ $N = xe^{xy} + 2y$

$\frac{\partial M}{\partial y} = e^{xy} + xy\,e^{xy}$ $\frac{\partial N}{\partial x} = e^{xy} + xye^{xy}$ — 3 M

equation is exact

sol is $\int ye^{xy}dx + \int 2y\,dy = c$ — 2 M

$\Rightarrow$ $e^{xy} + y^2 = c$ is reqd solution

3(a) Growth problem, $\frac{dN}{dt} \propto N$

$N = ce^{kt} \longrightarrow (1)$

Given $t = 0$ $N = 200$
$t = 3$ $N = 500$
$t = 6$ $N = ?$ — 2 M

put $t = 0$ in (1) $c = 200$ — 1 M

put $t = 3$ in (1) $k = \frac{1}{3}\ln(5/2)$ — 1 M

$\therefore$ $N = 200\,(5/2)^{t/3}$

At $t = 6$, $N = 200\,(5/2)^2 = 1250$ — 1 M

(b) $M = y\ln y$ $N = x - \ln y$

$\frac{\partial M}{\partial y} = 1 + \ln y$ $\frac{\partial N}{\partial x} = 1$ — 1 M

$\frac{-\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x}}{M} = \frac{-\ln y}{y\ln y} = \frac{-1}{y}$ is function of y alone

$I.F = e^{-\int \frac{1}{y}dy} = e^{-\ln y} = \frac{1}{y}$ — 2 M

$\ln y\,dx + \left(\frac{x}{y} - \frac{\ln y}{y}\right)dy = 0$ is exact

solution is $\int\limits_{y\ \text{const}} \ln y\,dx - \int \frac{\ln y}{y}\,dy = c$ — 2 M

$x\ln y - \frac{(\ln y)^2}{2} = c$ is reqd solution.

4

4 (a) Given $L = 0.25,\ R = 250,\ C = 2\times10^{-6}$

at $t=0 \quad q = 0.002$

$t = 0 \quad \frac{dq}{dt} = 0$

$$\frac{d^2q}{dt^2} + 1000\frac{dq}{dt} + 2\times10^{6} q = 0$$

Roots are $-500 \pm 500\sqrt{7}\, i$ ——— 2M

$$q = e^{-500t}\left[A\cos 500\sqrt{7}t + B\sin 500\sqrt{7}t\right] \rightarrow (1)$$

$$\frac{dq}{dt} = -500e^{-500t}\left[A\cos 500\sqrt{7}t + B\sin 500\sqrt{7}t\right]$$ ——— 2M

$$+ e^{-500t}\left(-500\sqrt{7}A\sin 500\sqrt{7}t + 500\sqrt{7}B\cos 500\sqrt{7}t\right) \rightarrow (2)$$

put $t=0$ in (1) $\quad A = 0.002$

put $t=0$ in (2) $\quad B = -0.0007$ ——— 1M

$$\therefore q = e^{-500t}\left[0.002\cos 500\sqrt{7}t - 0.0007\sin 500\sqrt{7}t\right]$$

4 (b) $(D^2+D+1)y = (1-e^x)^2$

$m = -1 \pm i\sqrt{3}/2$

$y_c = e^{-x}\left[C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x\right]$ ——— 2M

$$y_p = \frac{1}{D^2+D+1}(1-2e^x+e^{2x}) = 1 - \frac{2}{3}e^x + \frac{e^{2x}}{7}$$ ——— 3M

5 (a) $(D^3+1)y = \sin(2x+3)$

$$y_p = \frac{1}{D^3+1}\sin(2x+3) = \frac{1\,(\sin(2x+3))}{-4D+1}$$ ——— 2M

$$= \frac{(1+4D)}{1-16D^2}\sin(2x+3) = \frac{\sin(2x+3) + 8\cos(2x+3)}{65}$$ ——— 3M

(b) $(D^2+4)y = \tan 2x$

$y_c = C_1\cos 2x + C_2\sin 2x$

$$W = \begin{vmatrix}\cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x\end{vmatrix} = 2$$ ——— 2M

$$u_1 = -\int \frac{\tan 2x \times \sin 2x}{2}\,dx = -\frac{1}{4}\ln|\sec 2x + \tan 2x| + \frac{\sin 2x}{4}$$ ——— 1M

$$u_2 = \int \frac{\tan 2x \times \cos 2x}{2}\,dx = -\frac{\cos 2x}{4}$$ ——— 1M

$y_p = u_1 y_1 + u_2 y_2 = -\frac{\cos 2x}{4}\ln|\sec 2x + \tan 2x|$ ——— 1M

5

6(a) $(D^2-2DD'+D'^2)z = e^{x+y}$

$m^2-2m+1 = (m-1)^2 = 0, \ m = 1, 1$

general solution is $\phi_1(y+x) + x\phi_2(y+x)$ — 2M

particular integral is

$$\frac{1}{D^2-2DD'+D'^2} e^{x+y} = x\frac{1}{2D-2} e^{x+y} = \frac{x^2 e^{x+y}}{2} \text{ — 3M}$$

(b) $$\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{y^2-x^2}$$

$\frac{dx}{x} = \frac{dy}{-y} \Rightarrow \ln xy = \ln c_1 \Rightarrow c_1 = xy$ — 2M

$\frac{x\,dx+y\,dy}{x^2-y^2} = \frac{dz}{y^2-x^2} \Rightarrow \cancel{x\,dx+y\,dy} = -dz$ — 2M

$x^2/2 + y^2/2 + z = c_2, \ \phi(c_1, c_2) = 0$ — 1M

7(a) $$\frac{dx}{y^2z/x} = \frac{dy}{xz} = \frac{dz}{y^2} \text{ — 1M}$$

$\frac{x\,dx}{y^2z} = \frac{dy}{xz} \Rightarrow x^2dx - y^2dy = 0$

$\Rightarrow c_1 = x^3/3 - y^3/3$ — 2M

$\frac{x\,dx}{y^2z} = \frac{dz}{y^2} \Rightarrow x\,dx = z\,dz$

$c_2 = x^2/2 - z^2/2$ — 2M

$\phi(x^3-y^3, x^2-z^2) = 0$

(b)

$x^2+y^2+(z-a)^2 = r^2 \rightarrow (1)$ — 2M

Diff. w.r. to x, y

$2x + 2(z-a)p = 0$

$2y + 2(z-a)q = 0$

$(z-a) = -x/p$ — 1M

$(z-a) = -y/q$ — 1M

$x/p = y/q \Rightarrow py - qx = 0$ is required PDE

— 1M

8 (a) $\phi = xy^2 + yz^3$

$f = x\ln z - y^2 + 4 = 0$

$\bar{a} = \nabla f_{(-1,2,1)} = \left(\ln z\, i - 2y j + \frac{x}{z} k\right)_{(-1,2,1)}$

$= -4j - k$ ——— 2 M

$\nabla\phi = y^2 i + (2xy + z^3) j + 3yz^2 k$

$\nabla\phi_{(2,-1,1)} = i - 3j - 3k$ ——— 2 M

$\nabla\phi \cdot \frac{\bar{a}}{|\bar{a}|} = (i - 3j - 3k)\cdot\frac{(-4j-k)}{\sqrt{17}} = \frac{15}{\sqrt{17}}$ ——— 1 M

(b) $\mathrm{Div}(r^n \bar{r}) = \sum i \frac{\partial}{\partial x} \cdot \sum x r^n i$

$\frac{\partial r}{\partial x} = x/r \quad \frac{\partial r}{\partial y} = y/r \quad \frac{\partial r}{\partial z} = z/r$ ——— 2 M

$\nabla \cdot r^n \bar{r} = \sum\left[r^n + n r^{n-1} \cdot x \frac{\partial r}{\partial x}\right] i$

$= \sum\left[r^n + n r^{n-1} \times \frac{x^2}{r}\right] = 3r^n + nr^n$

$= (n+3) r^n$ ——— 3 M

9 a. $\nabla^2 r^m = \sum \frac{\partial^2}{\partial x^2} r^m$ ——— 1 M

$= \sum \frac{\partial}{\partial x}\left[m r^{m-1} \frac{\partial r}{\partial x}\right] = \sum\left[\frac{\partial}{\partial x} m r^{m-1} \cdot \frac{x}{r}\right]$

$= m\sum \frac{\partial}{\partial x}\left[r^{m-2} \cdot x\right] = \sum\left\{m\left[1 \cdot r^{m-2} + x(m-2) r^{m-3} \frac{\partial r}{\partial x}\right]\right\}$ ——— 2 M

$= \sum\left[m r^{m-2} + (m^2 - 2m) r^{m-2} x^2\right]$

$= r^{m-2}\left[3m + m^2 - 2m\right] = m(m+1) r^{m-2}$ ——— 2 M

(b) $\phi_1 = ax^2 - byz - (a+2)x, \quad \nabla\phi_1 = [2ax - (a+2)]\, i - bz j - by k$

$\phi_2 = 4x^2 y + z^3 + 4 \quad \nabla\phi_2 = 8xy i + (4x^2) j + 3z^2 k$

$\bar{a} = \nabla\phi_1 (1,-1,2) = (a-2) i - 2bj + bk$ ——— 1 M

$\bar{b} = \nabla\phi_2 (1,-1,2) = -8i + 4j + 12k$ ——— 1 M

orthogonal $\Rightarrow \bar{a}\cdot\bar{b} = 0 \Rightarrow -8a + 16 - 8b + 12b = 0$

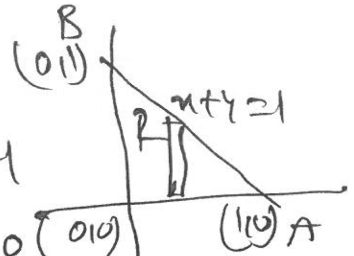
$\Rightarrow 8a - 4b = 16 \Rightarrow 4a - 2b = 8 \rightarrow (1)$

$(1,-1,2)$ lies on $ax^2 - byz = (a+2)x$ ——— 2 M

$a + 2b = a + 2 \Rightarrow b = 1$ ——— 1 M

put $b=1$ in (1) $a = 10/4$

$\boxed{a = 5/2,\ b = 1}$



10 (10) $M = 3x - 8y^2, \quad \frac{\partial M}{\partial y} = -16y$
$N = 4y - 6xy \quad \frac{\partial N}{\partial x} = -6y$ — 2M

Along OA, $y=0, dy=0$ | $\int_{OA} M\,dx + N\,dy = \int_0^1 3x\,dx = 3/2$ — 1M

Along AB, $y = 1-x$, $dy = -dx$ | $\int_{AB} M\,dx + N\,dy = \int_1^0 [-14x^2 + 29x - 12]\,dx = \frac{13}{6}$ — 1M

Along BO, $x = 0$, $dx = 0$ | $\int_{BO} M\,dx + N\,dy = \int_1^0 4y\,dy = -2$ — 1M

$\therefore \int_C M\,dx + N\,dy = \frac{3}{2} + \frac{13}{6} - 2 = \frac{9+13-12}{6} = \frac{10}{6} = \boxed{5/3}$ — 1M

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy = \int_{x=0}^{1}\int_{y=0}^{1-x} 10y\,dx\,dy = \boxed{5/3}$$ — 4M

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

Greens thm verified

(11) By GDT $\iint_S \bar{F}\cdot\bar{n}\,ds = \iiint_V \nabla\cdot\bar{F}\,dv$ — 2M

$\nabla\cdot\bar{F} = \Sigma i\frac{\partial}{\partial x}\cdot(2x^2y\,i - y^2 j + 4xz^2 k)$ — 2M

$= 4xy - 2y + 8xz$

$\iiint_V \nabla\cdot\bar{F}\,dv = \iiint_V (4xy - 2y + 8xz)\,dx\,dy\,dz$

Evaluating in cylindrical polar coordinates

$y = r\cos\theta \quad z = r\sin\theta$, x is 0 to 2 — 2M

$$\iiint_V \nabla\cdot\bar{F}\,dv = \int_{\theta=0}^{\pi/2}\int_{r=0}^{3}\int_{x=0}^{2}[4xr\cos\theta - 2r\cos\theta + 8xr\sin\theta]\,r\,dr\,d\theta$$

$$= \int_{\theta=0}^{\pi/2}\int_{r=0}^{3}[8r^2\cos\theta - 4r^2\cos\theta + 16r^2\sin\theta]\,dr\,d\theta$$

$$= \int_{\theta=0}^{\pi/2}\int_{r=0}^{3}(4r^2\cos\theta + 16r^2\sin\theta)\,dr\,d\theta$$

$= 36 + 144$

$= \boxed{180}$ — 4M

* Solution may be done in Cartesian coordinates.