PVP 23

Code: 23ME3603

III B.Tech - II Semester - Regular Examinations – APRIL 2026

FINITE ELEMENT METHODS
(MECHANICAL ENGINEERING)

Duration: 3 hours Max. Marks: 70

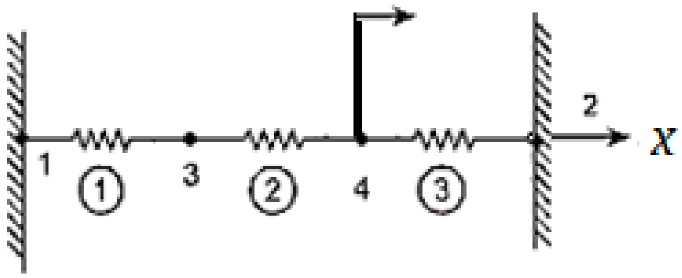
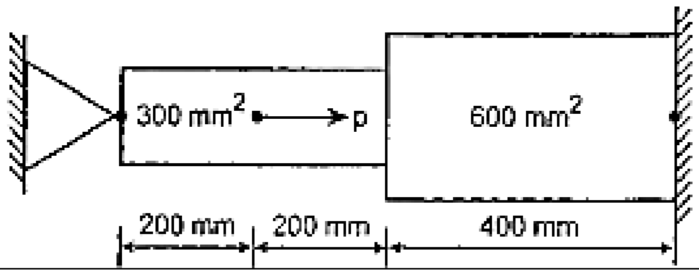
Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

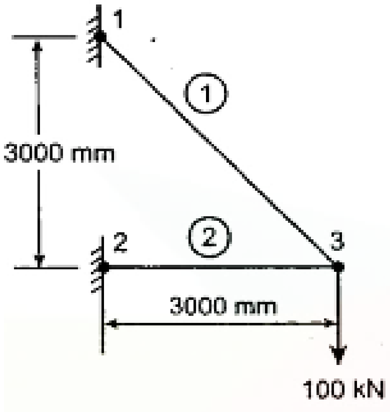
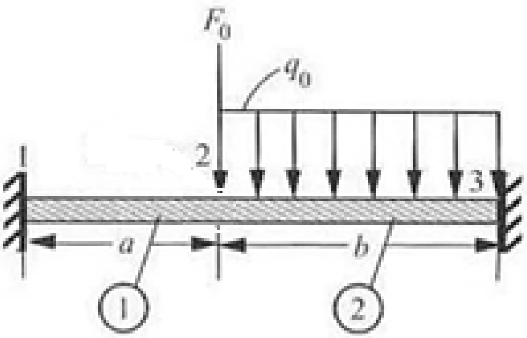
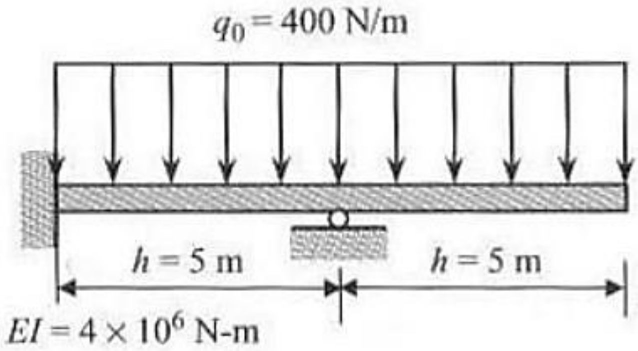
BL – Blooms Level CO – Course Outcome

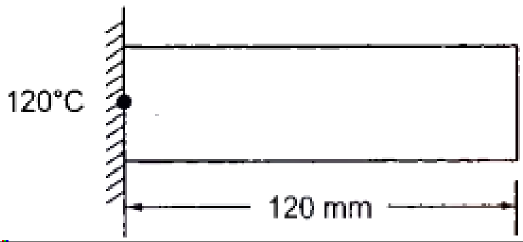
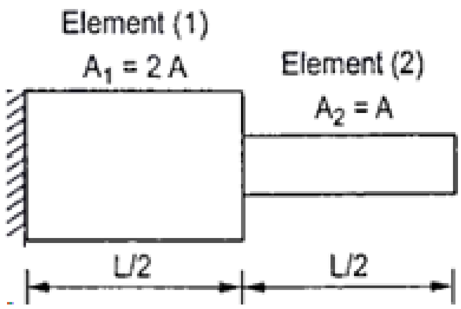
PART – A

		BL	CO
1.a)	Give examples of plane strain conditions.	L1	CO1
1.b)	Write statement for the principle of minimum potential energy.	L1	CO1
1.c)	What are the different 2-D elements available in FEM?	L2	CO2
1.d)	Write stiffness matrix for a 2-D truss element.	L2	CO2
1.e)	What is lumped load vector for 1-D beam element?	L2	CO3
1.f)	How is beam element different from 1-D bar element?	L2	CO3
1.g)	What is Constant Strain Triangular element (CST)?	L2	CO4
1.h)	What is the meaning of super-parametric formulation?	L1	CO4
1.i)	Write the general 1-D heat transfer equation.	L2	CO5
1.j)	What are possible boundary conditions for a fin problem?	L2	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	Derive the D matrix for 3-D stress strain relations from fundamentals.		L2	CO1	10 M
OR					
3	A spring assemblage with arbitrarily numbered nodes is shown in fig. The Nodes 1 and 2 are fixed and a force of 500kN is applied at node 4 in the x-direction. Calculate Nodal displacements. Take spring constants K_1 =100kN/m, K_2 = 200kN/m and K_3 = 300kN/m.  		L3	CO1	10 M
UNIT-II					
4	Consider the stepped bar shown in fig. Obtain Nodal displacements and Support reactions. Take E = $2x10^5 N/mm^2$. Load P = 400kN.  		L3	CO2	10 M
OR					
5	Find the unknown nodal displacements of the truss shown in fig. Take A_1 = $500mm^2$, A_2 =$1200mm^2$ and E = $2x10^5$ N/mm^2.		L3	CO2	10 M

				
UNIT-III				
6	Model the beam problem with two elements shown in fig. Find displacements at node 2. 	L4	CO3	10 M
OR				
7	Model the problem shown in fig. using two 1-D beam elements. Determine displacements at the nodes. 	L4	CO3	10 M
UNIT-IV				
8	Explain the axi-symmetric formulation in Finite Element Analysis. Also derive the strain–displacement relations for axi-symmetric elements and explain force vector in detail.	L2	CO4	10 M

OR					
9	a)	Compute the integral $\int_{-1}^{+1}\left(x^3+5x^2-6\right)dx$ using the Gaussian 2 point formula.	L3	CO4	7 M
	b)	Obtain Jacobian for 4-node quadrilateral element.	L3	CO4	3 M
UNIT-V					
10	Calculate the temperature distribution in a 1-D fin shown in the fig. The fin is rectangular in shape and is 120mm long, 40mm wide and 10mm thick. Assume that tip is insulated. Use two elements. Take k = 0.3 W/mm˚C, h = 1× 10^{-3}W/$m^{2˚}$C, T_∞ = 20˚C. 		L3	CO5	10 M
OR					
11	Find the natural frequency of longitudinal vibration for the stepped bar as shown in fig. Take L = 400mm, A= 600 mm^2, E = 2×10^5 N/mm^2, ρ = 0.8 × 10^{-4} kg/mm^3. 		L3	CO5	10 M

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Scheme – FEM – 23ME3603 – PVP23 Regulations – APRIL 2026

PART – A			
1	a	Any two examples/diagrams	2M
	b	Statement/Diagram	2M
	c	Triangular/Quadrilateral/diagram	2M
	d	Stiffness matrix	2M
	e	Lumped load matrix	2M
	f	Any one difference/diagram	2M
	g	Explanation/Diagram	2M
	h	Super parametric definition	2M
	i	Heat Transfer equation	2M
	j	Any one Boundary condition	2M
PART – B			
UNIT – I			
2		6 Strain equations	3M
		ε_x in terms of stress components	3M
		σ_x in terms of strain components	2M
		D Matrix	2M
OR			
3		Given data	2M
		Spring Diagram	1M
		Strain Energy	3M
		Work Potential	2M
		Partial differentiation	1M
		Finding the displacements	1M
UNIT – II			
4		3 Stiffness matrices (each 2 marks)	6M
		Global stiffness matrix/ Reduced matrix	2M
		Load matrix	1M
		Solution	1M
OR			
5		2 Stiffness matrices (each 3 marks)	6M
		Global stiffness matrix/ Reduced matrix	2M
		Load matrix	1M
		Solution	1M
UNIT – III			
6		Finite Element modelling	1M
		2 stiffness matrices (each 3 marks)	6M
		Load vector	1M
		Reduced matrix	1M
		Solution	1M

OR			
7		Finite Element modelling	1M
		2 stiffness matrices (each 3 marks)	6M
		Load vector	1M
		Reduced matrix	1M
		Solution	1M
UNIT – IV			
8		Axisymmetric definition	2M
		Axisymmetric diagram	2M
		Strain – displacement derivation	2M
		Force vector conversion	4M
OR			
9	a	2 Point formula	2M
		Calculation of F(x1),f(x2)	2M
		Substitution	2M
		Solution	1M
	b	J matrix	3M
UNIT – V			
10		Finite element modelling	1M
		2 stiffness matrices	4M
		2 h matrices	2M
		2 load matrices	1M
		Global stiffness and h matrices	1M
		Solution	1M
OR			
11		Finite element modelling	1M
		2 stiffness matrices	4M
		2 mass matrices	2M
		Global stiffness and mass matrices	1M
		K-λM equation	1M
		Solution	1M

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Solutions to FEM – 23ME3603 – PVP23 Regulations – APRIL 2026

PART – A

1.

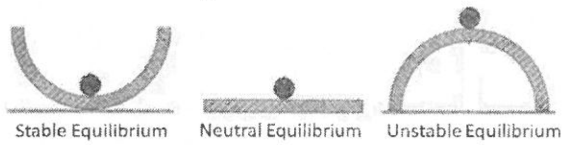
a.

- When the dimension of the body in the *z* direction is very large it is considered as plane strain problems
- In plane strain problem all the strain components in z directions are
- $\varepsilon_z = \gamma_{zx} = \gamma_{zy} = 0$ and strain components are specified by $\varepsilon_x, \varepsilon_y, \gamma_{xy}$
- There are many important problems of this kind for instance,
  - ✓ a retaining wall with lateral pressure
  - ✓ a culvert or tunnel
  - ✓ a cylindrical tube with internal pressure
  - ✓ a cylindrical roller compressed by forces in a diametral plane as in a roller bearing

b.

For conservative systems, of all the kinematically admissible displacement fields, those corresponding to equilibrium extremize the total potential energy. If the extreme condition is a minimum, the equilibrium state is stable.

In other words, for a stable equilibrium, the total potential energy must be minimum. This is Principle of minimum Potential Energy



The Total Potential Energy Π = Strain Energy + Work Potential

c. Triangular and Quadrilateral elements

d. $ke = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$

e. Lumped load vector means, load is concentrated at nodes only. Depending upon the load, the load vector is $\begin{bmatrix} \frac{q}{2} \\ 0 \\ \frac{q}{2} \\ 0 \end{bmatrix}$ or $\begin{bmatrix} \frac{qL}{2} \\ 0 \\ \frac{qL}{2} \\ 0 \end{bmatrix}$

f.

Bar element will have only axial loads

Beam element will have transverse and bending loads

Bar Element Each node will have only 1 DOF

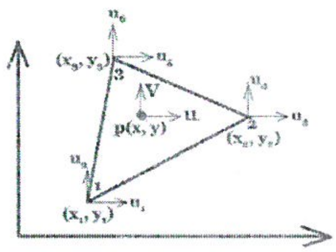
Beam element Each node will have 2 DOF

Bar Element

Beam Element

g.

- It is a triangular element
- Consisting of 3 nodes
- Each node have 2 dof and total 6 dof
- Strain is assumed to be constant throught the element



h.

- For super parametric elements Geometry uses Higher order shape functions and variables uses Lower order shape functions
- Order of shape functions for geometry > Order of shape functions for variables.

i. $\frac{d^2T}{dx^2} + \frac{Q}{K} = \frac{1}{\alpha}\frac{dT}{dt}$; Q is Internal heat generation, K Thermal conductivity α Thermal Diffusivity

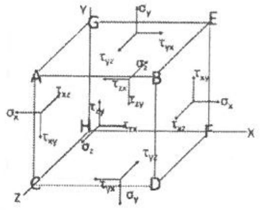
j.

- Temperature at nodes
- Heat incoming or outgoing

PART – B

UNIT – I

2.



According to Hook's law, the 3 linear and 3 shear strains are given as:

$$\epsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{\tau_{xy}}{\frac{E}{2(1+\mu)}} = \frac{2(1+\mu)}{E}\tau_{xy}\,.$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G} = \frac{2(1+\mu)}{E}\tau_{yz}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G} = \frac{2(1+\mu)}{E}\tau_{xz}$$

$$\begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \epsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{Bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\mu & -\mu & 0 & 0 & 0\\ -\mu & 1 & -\mu & 0 & 0 & 0\\ -\mu & -\mu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\mu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\mu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\mu)\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix}$$

$\boldsymbol{\varepsilon} = C\,\sigma$

$\sigma = C^{-1}\,\boldsymbol{\varepsilon}$

$\sigma = D\,\boldsymbol{\varepsilon}$

Hence, $C = \frac{1}{E}\begin{bmatrix}1 & -\mu & -\mu & 0 & 0 & 0\\ -\mu & 1 & -\mu & 0 & 0 & 0\\ -\mu & -\mu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0.5-\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 0.5-\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 0.5-\mu\end{bmatrix}$

Now, $\epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E}\left[\sigma_x - \mu(\sigma_y + \sigma_z) + \sigma_y - \mu(\sigma_x + \sigma_z) + \sigma_z - \mu(\sigma_x + \sigma_y)\,\right]$

$\Rightarrow \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E}\left[\sigma_x + \sigma_y + \sigma_z - \mu(\sigma_y + \sigma_z + \sigma_x + \sigma_z + \sigma_x + \sigma_y)\,\right]$

$\Rightarrow \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E}\left[\sigma_x + \sigma_y + \sigma_z - 2\mu(\sigma_x + \sigma_y + \sigma_z)\,\right]$

$\Rightarrow \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E}\left[(1-2\mu)(\sigma_x + \sigma_y + \sigma_z)\right]$ ...... (1)

But, we know that $\quad \epsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)]$

$\Rightarrow \mu(\sigma_y + \sigma_z) = [\sigma_x - E\epsilon_x]$

$\Rightarrow (\sigma_y + \sigma_z) = \frac{1}{\mu}[\sigma_x - E\epsilon_x]$

Now, from eq 1,

$$\Rightarrow \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E}\left[(1-2\mu)(\sigma_x + \frac{1}{\mu}[\sigma_x - E\epsilon_x])\right]$$

$$\Rightarrow E[\epsilon_x + \epsilon_y + \epsilon_z] = \sigma_x\,(1-2\mu) + \frac{\sigma_x\,(1-2\mu)}{\mu} - \frac{E\epsilon_x(1-2\mu)}{\mu}$$

$$\Rightarrow E[\epsilon_x + \epsilon_y + \epsilon_z] + \frac{E\epsilon_x(1-2\mu)}{\mu} = \sigma_x\,(1-2\mu)[\,1+\frac{1}{\mu}]$$

$$\Rightarrow E[\epsilon_x(1+\frac{(1-2\mu)}{\mu}) + \epsilon_y + \epsilon_z] = \sigma_x\,\frac{(1-2\mu)(1+\mu)}{\mu}$$

$$\Rightarrow \frac{E}{\mu}\left[\epsilon_x(1-\mu) + \mu\epsilon_y + \mu\epsilon_z\right] = \sigma_x\,\frac{(1-2\mu)(1+\mu)}{\mu}$$

$$\Rightarrow \boldsymbol{\sigma_x} = \frac{E}{(1-2\mu)(1+\mu)}[\epsilon_x(1-\mu) + \mu\epsilon_y + \mu\epsilon_z] \text{ similarly}$$

$$\Rightarrow \boldsymbol{\sigma_y} = \frac{E}{(1-2\mu)(1+\mu)}[\mu\epsilon_x + (1-\mu)\epsilon_y + \mu\epsilon_z]$$

$$\Rightarrow \boldsymbol{\sigma_z} = \frac{E}{(1-2\mu)(1+\mu)}[\mu\epsilon_x + \mu\epsilon_y + (1-\mu)\epsilon_z]$$

And also Shear stress $\tau_{xy} = G\gamma_{xy}$

$$\Rightarrow \tau_{xy} = \frac{E}{2(1+\mu)}\gamma_{xy}$$

$$\Rightarrow \tau_{xy} = \frac{E(1-2\mu)}{2(1+\mu)(1-2\mu)}\gamma_{xy}$$

$$\Rightarrow \tau_{xy} = \frac{E}{(1+\mu)(1-2\mu)}[\frac{(1-2\mu)}{2}]\gamma_{xy}$$

$$\Rightarrow \boldsymbol{\tau_{xy}} = \frac{E}{(1+\mu)(1-2\mu)}[0.5 - \mu]\gamma_{xy} \quad \text{similarly}$$

$$\Rightarrow \boldsymbol{\tau_{yz}} = \frac{E}{(1+\mu)(1-2\mu)}[0.5 - \mu]\gamma_{yz}$$

$$\Rightarrow \boldsymbol{\tau_{xz}} = \frac{E}{(1+\mu)(1-2\mu)}[0.5 - \mu]\gamma_{xz}$$

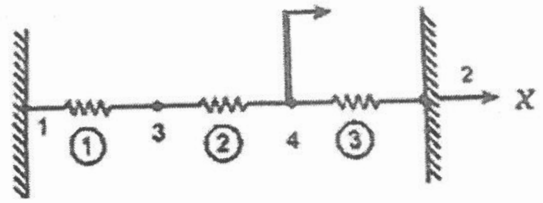
$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{Bmatrix} = \frac{E}{(1-2\mu)(1+\mu)}\begin{bmatrix}1-\mu & \mu & \mu & 0 & 0 & 0\\ \mu & 1-\mu & \mu & 0 & 0 & 0\\ \mu & \mu & 1-\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 0.5-\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 0.5-\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 0.5-\mu\end{bmatrix}\begin{Bmatrix}\epsilon_x\\ \epsilon_y\\ \epsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{Bmatrix}$$

Hence The for a 3D stress analysis

The D matrix is $D = \frac{E}{(1-2\mu)(1+\mu)}\begin{bmatrix}1-\mu & \mu & \mu & 0 & 0 & 0\\ \mu & 1-\mu & \mu & 0 & 0 & 0\\ \mu & \mu & 1-\mu & 0 & 0 & 0\\ 0 & 0 & 0 & 0.5-\mu & 0 & 0\\ 0 & 0 & 0 & 0 & 0.5-\mu & 0\\ 0 & 0 & 0 & 0 & 0 & 0.5-\mu\end{bmatrix}$

(OR)

3.



Let q_1, and q_2 are the displacements of nodes 3 and 4

Let x_1, x_2 and x_3 are displacements of springs

$x_1 = q_1$; $x_2 = q_1 - q_2$; $x_3 = q_2$

Strain Energy of spring k_1 $SE_1 = \frac{1}{2} k x_1^2$

$SE_1 = \frac{1}{2} k_1 q_1^2$; $SE_2 = \frac{1}{2} k_2 (q_1 - q_2)^2$; $SE_3 = \frac{1}{2} k_3 (q_2)^2$

Work Potential

Due to $F_1 = - P. q_2$;

The Total Potential Energy Π = Strain Energy + Work Potential

$\Pi = \frac{1}{2} k_1 q_1^2 + \frac{1}{2} k_2 (q_1 - q_2)^2 + \frac{1}{2} k_3 q_2^2 + - P.q_2$

For the potential energy minimum

$$\frac{\partial \pi}{\partial q_1} = 0$$

$\Rightarrow K_1 q_1 + K_2 (q_1 - q_2) + = 0$

$\Rightarrow (K_1 + K_2) q_1 - K_2 q_2 = 0$ .... (1)

$$\frac{\partial \pi}{\partial q_2} = 0$$

$\Rightarrow - K_2 (q_1 - q_2) + K_3 q_2 - P = 0$

$\Rightarrow - K_2 q_1 + (K_2 + K_3) q_2 = P$ .... (2)

The Two equations are

$(K_1 + K_2) q_1 - K_2 q_2 = 0$ .... (1)

$- K_2 q_1 + (K_2 + K_3) q_2 = P$ .... (2)

By substituting the given values in to these equations

$\Rightarrow 300 q_1 - 200 q_2 = 0$ ...(3)

$\Rightarrow - 200 q_1 + 500 q_2 = 500 \times 10^3$ ... (4)

By solving above two equations we get

$q_1 = 909.1$mm $q_2 = 1363.63$ mm Ans...

UNIT – II

4. The equivalent finite element model is as follows

Element	1	2	3
Node numbers	1, 2	2, 3	3, 4
Length L	200 mm	200 mm	400 mm
Area A	300 mm^2	300 mm^2	600 mm^2
Young's modulus E	$2x10^5$ N/mm^2	$2x10^5$ N/mm^2	$2x10^5$ N/mm^2

Element stiff ness matrix for element 1

$$k_1 = \frac{A_1E_1}{L_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_1 = \frac{(300)(2x10^5)}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_1 = 3x10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

1 2

$$k_1 = \begin{bmatrix} 3x10^5 & -3x10^5 \\ -3x10^5 & 3x10^5 \end{bmatrix}\begin{matrix} 1 \\ 2 \end{matrix}$$

Element stiff ness matrix for element 2

$$k_2 = \frac{A_2E_2}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = \frac{(300)(2x10^5)}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_2 = 3x10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

2 3

$$k_2 = \begin{bmatrix} 3x10^5 & -3x10^5 \\ -3x10^5 & 3x10^5 \end{bmatrix}\begin{matrix} 2 \\ 3 \end{matrix}$$

Element stiff ness matrix for element 3

$$k_3 = \frac{A_3E_3}{L_3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_3 = \frac{(600)(200x10^3)}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$k_3 = 3x10^5\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

3 4

$$k_3 = \begin{bmatrix} 3x10^5 & -3x10^5 \\ -3x10^5 & 3x10^5 \end{bmatrix}\begin{matrix} 3 \\ 4 \end{matrix}$$

The Global Stiffness matrix is

$K = k_1 + k_2 + k_3$

1 2 3 4

$$K = \begin{bmatrix} 3x10^5 & -3x10^5 & 0 & 0 \\ -3x10^5 & 6x10^5 & -3x10^5 & 0 \\ 0 & -3x10^5 & 6x10^5 & -3x10^5 \\ 0 & 0 & -3x10^5 & 3x10^5 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix}$$

$$\text{Load vector } F = \begin{bmatrix} 0 \\ 400x10^3 \\ 0 \\ 0 \end{bmatrix}$$

Hence, the final equation [K]{u} = [F]

$$\begin{bmatrix} 3x10^5 & -3x10^5 & 0 & 0 \\ -3x10^5 & 6x10^5 & -3x10^5 & 0 \\ 0 & -3x10^5 & 6x10^5 & -3x10^5 \\ 0 & 0 & -3x10^5 & 3x10^5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 400x10^3 \\ 0 \\ 0 \end{bmatrix}$$

By applying the boundary conditions are $u_1 = 0$ and $u_4 = 0$ we will get

$$\begin{bmatrix} 3x10^5 & -3x10^5 & 0 & 0 \\ -3x10^5 & 6x10^5 & -3x10^5 & 0 \\ 0 & -3x10^5 & 6x10^5 & -3x10^5 \\ 0 & 0 & -3x10^5 & 3x10^5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} 0 \\ 400x10^3 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6x10^5 & -3x10^5 \\ -3x10^5 & 6x10^5 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 400x10^3 \\ 0 \end{bmatrix}$$

$\Rightarrow 6x10^5 u_2 - 3x10^5 u_3 = 400x10^3$

$-3x10^5 u_2 + 6x10^5 u_3 = 0$

<u>Nodal displacements</u>

On solving above equations we will get $u_2 = 0.888$mm and $u_3 = 0.444$ mm

<u>Reaction at Node 1:</u>

$R_1 = K^1 u - F_1$

$$R_1 = [3x10^5 \quad -3x10^5 \quad 0 \quad 0] \begin{Bmatrix} 0 \\ 0.888 \\ 0.444 \\ 0 \end{Bmatrix} - [0]$$

$\Rightarrow R_1 = -266.64$ kN

<u>Reaction at Node 4:</u>

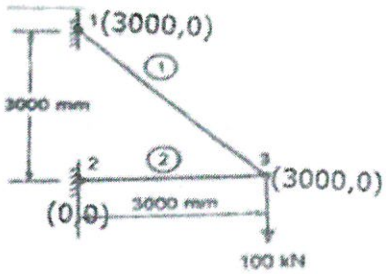
$R_4 = K^4 u - F_4$

$$R_4 = [0 \quad 0 \quad -3x10^5 \quad 3x10^5] \begin{Bmatrix} 0 \\ 0.888 \\ 0.444 \\ 0 \end{Bmatrix} - [0]$$

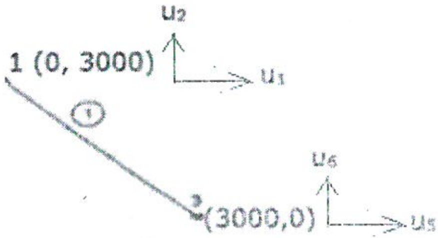
$\Rightarrow R_4 = -133.2$ kN

(OR)

5.



Element – 1:



Area A = 500 mm^2

$x_1 = 3000$, $x_2 = 0$, $y_1 = 0$, $y_2 = 3000$

Length of the element $L=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 3000)^2 + (3000 - 0)^2} =$ 4242.64 mm

$l = \cos\theta = \frac{x_2 - x_1}{L} = \frac{0-3000}{4242.64} = -0.707$

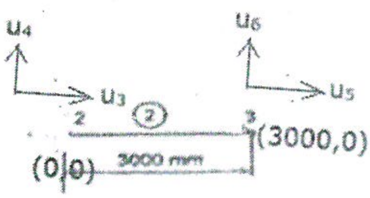
$m = \sin\theta = \frac{y_2 - y_1}{L} = \frac{3000-0}{4242.64} = 0.707$

Element Stiffness matrix

$$k_{e1} = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} = \frac{500x2x10^5}{4242.64}\begin{bmatrix} 0.5 & -0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$k_{e1} = \begin{bmatrix} 1 & 2 & 5 & 6 \\ 11.79x10^3 & -11.79x10^3 & -11.79x10^3 & 11.79x10^3 \\ -11.79x10^3 & 11.79x10^3 & 11.79x10^3 & -11.79x10^3 \\ -11.79x10^3 & 11.79x10^3 & 11.79x10^3 & -11.79x10^3 \\ 11.79x10^3 & -11.79x10^3 & -11.79x10^3 & 11.79x10^3 \end{bmatrix}\begin{matrix} \\ 1 \\ 2 \\ 5 \\ 6 \end{matrix}$$

Element – 2:



Area A = 1200 mm^2

$x_1 = 0$, $x_2 = 3000$, $y_1 = 0$, $y_2 = 0$

Length of the element L=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ =$\sqrt{(3000-)^2 + (0-0)^2}$ = 3000 mm

l = cos $\boldsymbol{\theta}$ = $\frac{x_2 - x_1}{L} = \frac{3000-0}{3000} = 1$

m = sin $\boldsymbol{\theta}$ = $\frac{y_2 - y_1}{L} = \frac{0-0}{3000} = 0$

Element Stiffness matrix

$$k_{e2} = \frac{AE}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} = \frac{1200x2x10^5}{3000}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k_{e2} = \begin{bmatrix} 3 & 4 & 5 & 6 \\ 80x10^3 & 0 & -80x10^3 & 0 \\ 0 & 0 & 0 & 0 \\ -80x10^3 & 0 & 80x10^3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$K = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 11.79x10^3 & -11.79x10^3 & 0 & 0 & -11.79x10^3 & -11.79x10^3 \\ -11.79x10^3 & 11.79x10^3 & 0 & 0 & 11.79x10^3 & -11.79x10^3 \\ 0 & 0 & 91.79x10^3 & 0 & -80x10^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -11.79x10^3 & 23.57x10^3 & -80x10^3 & 0 & 91.79x10^3 & -11.79x10^3 \\ 11.79x10^3 & -11.79x10^3 & 0 & 0 & -11.79x10^3 & 11.79x10^3 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -100x10^3 \end{bmatrix}$$

The reduced matrix

$$\begin{bmatrix} 91.79x10^3 & -11.79x10^3 \\ -11.79x10^3 & 11.79x10^3 \end{bmatrix} \begin{Bmatrix} u_5 \\ u_6 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -100x10^3 \end{bmatrix}$$

$91.79x10^3 u_5 - 11.79x10^3 u_6 = 0$

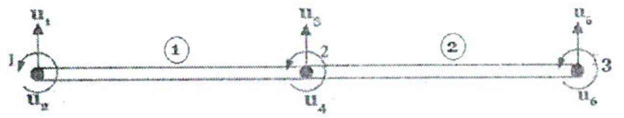
$-11.79x10^3 u_5 + 11.79x10^3 u_6 = -100x10^3$

By solving we will get $u_5 = -1.25$mm and $u_6 = -9.73$mm

UNIT – III

6.

The given problem can be modelled with 2 beam elements



Element	1	2
Nodes	1, 2	2, 3
Displacements	u_1, u_2, u_3, u_4	$u_3, u_4, u_5, u_6,$
Length	a	b
EI	EI	EI

$$k_1 = \frac{EI}{a^3}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 12 & 6a & -12 & 6a \\ 6a & 4a^2 & -6a & 2a^2 \\ -12 & -6a & 12 & -6a \\ 6a & 2a^2 & -6a & 4a^2 \end{bmatrix}\begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}, \quad k_2 = \frac{EI}{b^3}\begin{bmatrix} 3 & 4 & 5 & 6 \\ 12 & 6b & -12 & 6b \\ 6b & 4b^2 & -6b & 2b^2 \\ -12 & -6b & 12 & -6b \\ 6b & 2b^2 & -6b & 4b^2 \end{bmatrix}\begin{matrix} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$F_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \qquad F_2 = \begin{bmatrix} -(F_0 + \frac{q_0 L}{2}) \\ \frac{q_0 L^2}{12} \\ -\frac{q_0 L}{2} \\ -\frac{q_0 L^2}{12} \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix} \qquad \text{and } F = \begin{bmatrix} 0 \\ 0 \\ -(F_0 + \frac{q_0 L}{2}) \\ \frac{q_0 L^2}{12} \\ -\frac{q_0 L}{2} \\ -\frac{q_0 L^2}{12} \end{bmatrix}$$

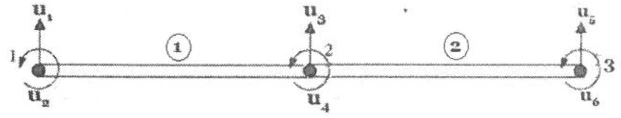
$$K = \begin{bmatrix} 3 & 4 \\ \frac{12\,EI}{a^3} + \frac{12\,EI}{b^3} & -\frac{6\,EI}{a^2} + \frac{6\,EI}{b^2} \\ -\frac{6\,EI}{a^2} + \frac{6\,EI}{b^2} & \frac{4\,EI}{a} + \frac{4\,EI}{b} \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}\begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} -(F_0 + \frac{q_0 L}{2}) \\ \frac{q_0 L^2}{12} \end{bmatrix}$$

$$\begin{bmatrix} \frac{12\,EI}{a^3} + \frac{12\,EI}{b^3} & -\frac{6\,EI}{a^2} + \frac{6\,EI}{b^2} \\ -\frac{6\,EI}{a^2} + \frac{6\,EI}{b^2} & \frac{4\,EI}{a} + \frac{4\,EI}{b} \end{bmatrix}\begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} -(F_0 + \frac{q_0 L}{2}) \\ \frac{q_0 L^2}{12} \end{bmatrix}$$

(OR)

7.

The given problem can be modelled with 2 beam elements



Element	1	2
Nodes	1, 2	2, 3
Displacements	u_1, u_2, u_3, u_4	$u_3, u_4, u_5, u_6,$
Length	5000	5000
EI	$4x10^9$	$4x10^9$

$$k_1 = \frac{4x10^9}{(5000)^3}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 12 & 30000 & -12 & 3000 \\ 3000 & 1x10^8 & -30000 & 0.5x10^8 \\ -12 & -30000 & 12 & -30000 \\ 30000 & 0.5x10^8 & -30000 & 1x10^8 \end{bmatrix}\begin{matrix} \\ 1 \\ 2, \\ 3 \\ 4 \end{matrix}$$

$$k_2 = \frac{4x10^9}{(5000)^3}\begin{bmatrix} 3 & 4 & 5 & 6 \\ 12 & 30000 & -12 & 3000 \\ 3000 & 1x10^8 & -30000 & 0.5x10^8 \\ -12 & -30000 & 12 & -30000 \\ 30000 & 0.5x10^8 & -30000 & 1x10^8 \end{bmatrix}\begin{matrix} \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$F_1 = \begin{bmatrix} -1000 \\ 8.33x10^5 \\ -1000 \\ -8.33x10^5 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \qquad F_1 = \begin{bmatrix} -1000 \\ 8.33x10^5 \\ -1000 \\ -8.33x10^5 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$F = \begin{bmatrix} -1000 \\ 8.33x10^5 \\ -2000 \\ -0 \\ -1000 \\ -8.33x10^5 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$K = \begin{bmatrix} 6.4x10^6 & -960 & 1.6x10^6 \\ -960 & 0.384 & -960 \\ 1.6x10^6 & -960 & 3.2x10^6 \end{bmatrix}\begin{Bmatrix} u_4 \\ u_5 \\ u_6 \end{Bmatrix} = \begin{bmatrix} -0 \\ -1000 \\ -8.33x10^5 \end{bmatrix}$$

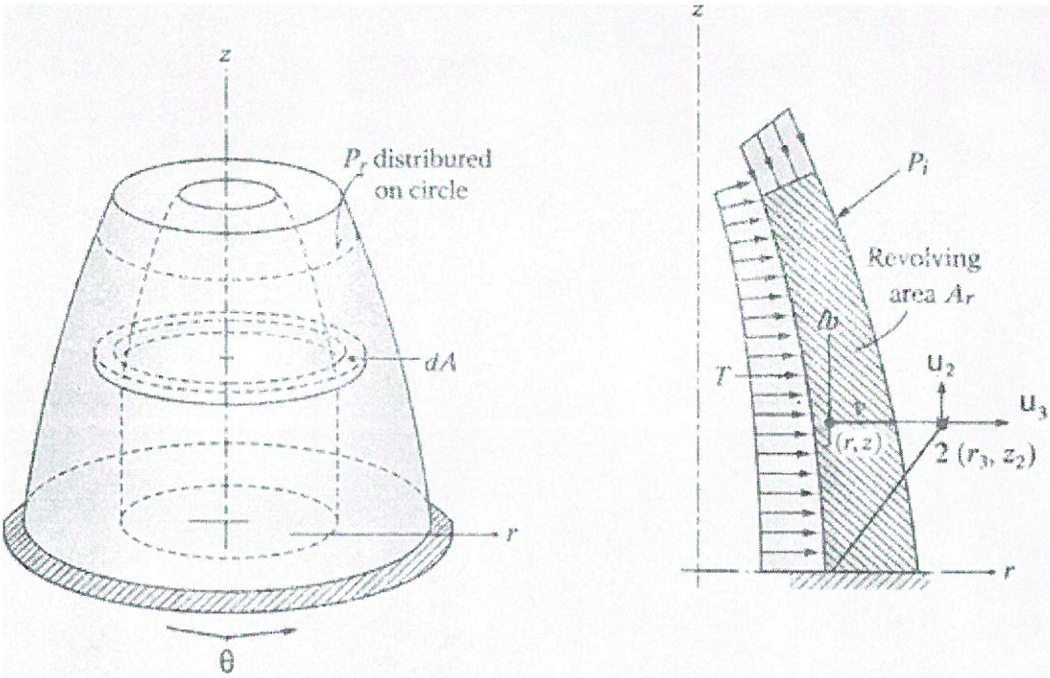
$u_4 = 1.04x10^{-4}$, $u_5 = 2.60mm$, $u_6 = 6.94x10^{-4}$

UNIT – IV

8.

Axisymmetric Element:

- Problems involving three-dimensional axisymmetric solids or solids of revolution, subjected to axisymmetric loading can be reduced to two-dimensional problems
- With reference to figure, the deformations, stresses are independent of angle $\boldsymbol{\theta}$, and it is a two-dimensional problem in r-z coordinates.



- The axisymmetric problem is viewed as two-dimensional problem; we will use an element similar to triangular element can be used to solve such problems. This element is known as axisymmetric triangular element.
- The dof at each node is 2.
- For the axisymmetric triangular element

$$\text{Strain matrix } \varepsilon = \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \gamma_{rz} \\ \varepsilon_\theta \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial r} \\ \dfrac{\partial v}{\partial z} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial v}{\partial r} \\ \dfrac{u}{r} \end{Bmatrix}$$

$$\text{Stress matrix } \sigma = \begin{Bmatrix} \sigma_r \\ \sigma_z \\ \tau_{rz} \\ \sigma_\theta \end{Bmatrix}$$

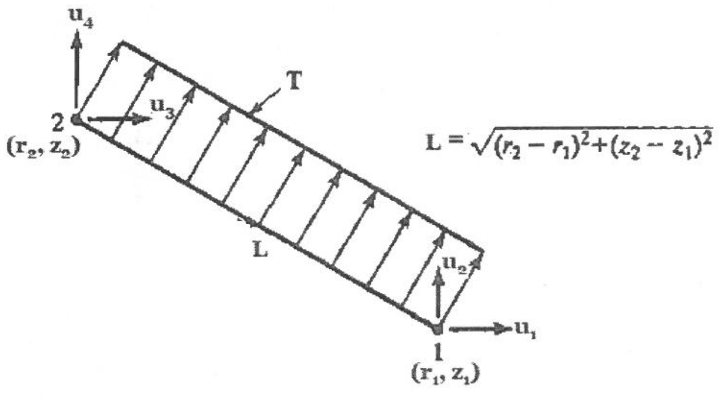
- For the axisymmetric triangular element:

$$D = \frac{E(1-\mu)}{(1-2\mu)(1+\mu)} \begin{bmatrix} 1 & \frac{\mu}{1-\mu} & 0 & \frac{\mu}{1-\mu} \\ \frac{\mu}{1-\mu} & 1 & 0 & \frac{\mu}{1-\mu} \\ 0 & 0 & \frac{1-2\mu}{2(1-\mu)} & 0 \\ \frac{\mu}{1-\mu} & \frac{\mu}{1-\mu} & 0 & 1 \end{bmatrix}$$

- Shape functions $N_1 = N_2 = N_3 = \frac{1}{3}$
- $J = \begin{bmatrix} r_{13} & z_{13} \\ r_{23} & z_{23} \end{bmatrix}$
- det J = 2 x Area of triangle
- $B = \begin{bmatrix} \frac{z_{23}}{\det J} & 0 & \frac{z_{31}}{\det J} & 0 & \frac{z_{12}}{\det J} & 0 \\ 0 & \frac{r_{32}}{\det J} & 0 & \frac{r_{13}}{\det J} & 0 & \frac{r_{21}}{\det J} \\ \frac{r_{32}}{\det J} & \frac{z_{23}}{\det J} & \frac{r_{13}}{\det J} & \frac{z_{31}}{\det J} & \frac{r_{21}}{\det J} & \frac{z_{12}}{\det J} \\ \frac{N_1}{r} & 0 & \frac{N_2}{r} & 0 & \frac{N_3}{r} & 0 \end{bmatrix}$
- **Stiffness matrix k = 2ΠrA B^T D B**, where $r = \frac{r_1+r_2+r_3}{3}$

<u>Force Vector:</u>

- Body Force vector $f = \frac{2\pi rA}{3} \begin{Bmatrix} f_r \\ f_z \\ f_r \\ f_z \\ f_r \\ f_z \end{Bmatrix}$
- Tractive Force



Now, let $C = \frac{z_2-z_1}{L}$ and $S = \frac{r_1-r_2}{L}$

$T_r = -CT$; $T_z = -ST$; $a = \frac{2r_1+r_2}{6}$ and $b = \frac{r_1+2r_2}{6}$

$$T = 2\pi L \begin{bmatrix} aT_r \\ aT_z \\ bT_r \\ bT_z \end{bmatrix}$$

(OR)

9.

$\int_{-1}^{+1}[\mathbf{x^3 + 5x^2 - 6}]\mathbf{dx}$

Sol:

I = $\int_{-1}^{+1}[x^3 + 5x^2 - 6]dx$

f(x) = $[x^3 + 5x^2 - 6]$

$w_1 = 1$ and $w_2 = 1$

$x_1 = 0.57735$ and $x_2 = -0.57735$

$f(x_1) = [0.57735^3 + 5(0.57735)^2 - 6] = -4.14$

$f(x_2) = [(-0.57735)^3 + 5(-0.57735)^2 - 6] = -4.526$

$\int_{-1}^{+1}\left[3e^x + x^2 + \frac{1}{(x+2)}\right]dx = w_1 f(x_1) + w_2 f(x_2) = 1(-4.14)+1(-4.526) = -8.666$

(b)

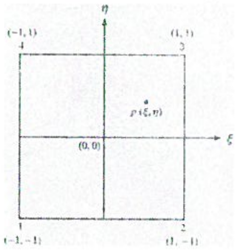
Quadrilateral element is and isoparametric element. That means, the same shape functions can be used to express the coordinates of any point in side the element (ex. p (x, y)).

That means

$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4$

$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$

In order to simplify the process, let us consider a quadrilateral element with local coordinates ξ and η.



The four shape functions can be written as

$N_1 = \frac{1}{4}(1-\xi)(1-\eta)$, $N_2 = \frac{1}{4}(1+\xi)(1-\eta)$, $N_3 = \frac{1}{4}(1+\xi)(1+\eta)$, $N_4 = \frac{1}{4}(1-\xi)(1+\eta)$

The strain components in 2D stress analysis is

$\varepsilon_x = \frac{\partial u}{\partial x}, \varepsilon_y = \frac{\partial v}{\partial y}, \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

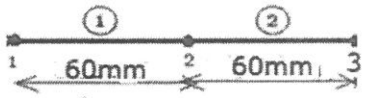
$\frac{\partial u}{\partial \xi} = \frac{\partial u}{\partial x}.\frac{\partial x}{\partial \xi} + \frac{\partial u}{\partial y}.\frac{\partial y}{\partial \xi}$ and $\frac{\partial u}{\partial \eta} = \frac{\partial u}{\partial x}.\frac{\partial x}{\partial \eta} + \frac{\partial u}{\partial y}.\frac{\partial y}{\partial \eta}$

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{Bmatrix}$$

Where J is Jacobian matrix $[J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$

UNIT – V

10.



Element	1	2
Nodes	1, 2	2, 3
Thermal conductivity	0.3	0.3
Length	60	60
Thermal convective	$1x10^{-9}$	$1x10^{-9}$
Thickness	10	10

<u>Element – 1</u>

$$ke_1 = \frac{K}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{0.3}{60}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 5x10^{-3} & -5x10^{-3}\\ -5x10^{-3} & 5x10^{-3}\end{bmatrix}\begin{matrix}\\ 1\\ 2\end{matrix}$$

$$he_1 = \frac{hL}{3t}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} = \frac{1x10^{-9}(60)}{3(10)}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4\,x10^{-9} & 2\,x10^{-9}\\ 2\,x10^{-9} & 4\,x10^{-9}\end{bmatrix}\begin{matrix}\\ 1\\ 2\end{matrix}$$

$$[R] = \frac{hT_\infty L}{t}\begin{Bmatrix}1\\ 1\end{Bmatrix} = \frac{(1x10^{-9})(20)(60)}{10}\begin{Bmatrix}1\\ 1\end{Bmatrix} = \begin{Bmatrix}1.2\,x10^{-7}\\ 1.2\,x10^{-7}\end{Bmatrix}\begin{matrix}1\\ 2\end{matrix}$$

<u>Element – 2</u>

$$Ke_2 = \frac{K}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \frac{0.3}{60}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5x10^{-3} & -5x10^{-3}\\ -5x10^{-3} & 5x10^{-3}\end{bmatrix}\begin{matrix}\\ 2\\ 3\end{matrix}$$

$$he_2 = \frac{hL}{3t}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} = \frac{1x10^{-9}(60)}{3(10)}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4\,x10^{-9} & 2\,x10^{-9}\\ 2\,x10^{-9} & 4\,x10^{-9}\end{bmatrix}\begin{matrix}\\ 2\\ 3\end{matrix}$$

$$[R] = \frac{hT_\infty L}{t}\begin{Bmatrix}1\\ 1\end{Bmatrix} = \frac{(1x10^{-9})(20)(60)}{10}\begin{Bmatrix}1\\ 1\end{Bmatrix} = \begin{Bmatrix}1.2\,x10^{-7}\\ 1.2\,x10^{-7}\end{Bmatrix}\begin{matrix}2\\ 3\end{matrix}$$

Global K matrix

$$K = \begin{bmatrix} 1 & 2 & 3\\ 5x10^{-3} & -5x10^{-3} & 0\\ -5x10^{-3} & 10x10^{-3} & -5x10^{-3}\\ 0 & -5x10^{-3} & 5x10^{-3}\end{bmatrix}\begin{matrix}\\ 1\\ 2\\ 3\end{matrix}$$

Global H matrix

$$H = \begin{bmatrix} 1 & 2 & 3\\ 4\,x10^{-9} & 2\,x10^{-9} & 0\\ 2\,x10^{-9} & 8\,x10^{-9} & 2\,x10^{-9}\\ 0 & 2\,x10^{-9} & 4\,x10^{-9}\end{bmatrix}\begin{matrix}\\ 1\\ 2\\ 3\end{matrix}$$

Global R Matrix

$$R = \begin{bmatrix}1.2\,x10^{-7}\\ 2.4\,x10^{-7}\\ 1.2\,x10^{-7}\end{bmatrix}$$

[K+H]{T}=[R]

$$\begin{bmatrix} 5x10^{-3} & -5x10^{-3} & 0 \\ -5x10^{-3} & 10x10^{-3} & -5x10^{-3} \\ 0 & -5x10^{-3} & 5x10^{-3} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 1.2\,x10^{-7} \\ 2.4\,x10^{-7} \\ 1.2\,x10^{-7} \end{bmatrix}$$

$T_1 = 120$

$$\begin{bmatrix} 10x10^{-3} & -5x10^{-3} \\ -5x10^{-3} & 5x10^{-3} \end{bmatrix} \begin{Bmatrix} T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 2.4\,x10^{-7} - (-5x10^{-3})(120) \\ 1.2\,x10^{-7} - 0 \end{bmatrix}$$

$$\begin{bmatrix} 10x10^{-3} & -5x10^{-3} \\ -5x10^{-3} & 5x10^{-3} \end{bmatrix} \begin{Bmatrix} T_2 \\ T_3 \end{Bmatrix} = \begin{bmatrix} 0.6 \\ 1.2\,x10^{-7} \end{bmatrix}$$

$T_2 = 33$ and $T_3 = 23$

(OR)

11

Density $\rho = 0.8x10^{-4}$ kg/mm^3

Elements	1	2
Nodes	1, 2	2, 3
Area (mm^2)	1200	600
Youngs Modulus (MPa)	$2x10^5$	$2x10^5$
Length (mm)	200	200
Density ρ	$0.8x10^{-4}$	$0.8x10^{-4}$

<u>Element – 1:</u>

Stiffness matrix $ke_1 = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\Rightarrow ke_1 = \frac{1200(2x10^5)}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$\Rightarrow ke_1 = \begin{bmatrix} 1 & 2 \\ 12x10^5 & -12x10^5 \\ -12x10^5 & 12x10^5 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \end{matrix}$$

Mass Matrix $me_1 = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow me_1 = \frac{0.8x10^{-4}(1200)(200)}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$\Rightarrow me_1 = \begin{bmatrix} 1 & 2 \\ 6.4 & 3.2 \\ 3.2 & 6.4 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \end{matrix}$$

<u>Element – 2:</u>

Stiffness matrix $ke_1 = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$\Rightarrow ke_1 = \frac{600(2x10^5)}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow ke_1 = \begin{bmatrix} 1 & 2 \\ 6\,x10^5 & -6x10^5 \\ -6x10^5 & 6x10^5 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \end{matrix}$$

Mass Matrix $me_1 = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$\Rightarrow me_1 = \frac{0.8x10^{-4}(600)(200)}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow me_1 = \begin{bmatrix} 1 & 2 \\ 3.2 & 1.6 \\ 1.6 & 3.2 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \end{matrix}$$

Global Matrices

$$K = \begin{bmatrix} 1 & 2 & 3 \\ 12x10^5 & -12x10^5 & 0 \\ -12x10^5 & 18x10^5 & -6x10^5 \\ 0 & -6x10^5 & 6x10^5 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix}$$

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 6.4 & 3.2 & 0 \\ 3.2 & 9.6 & 1.6 \\ 0 & 1.6 & 3.2 \end{bmatrix} \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix}$$

$|K - \lambda M|U = 0$ and We know that $U_1 = 0$

$$\begin{vmatrix} 18x10^5 - 9.6\lambda & -6x10^5 - 1.6\lambda \\ -6x10^5 - 1.6\lambda & 6x10^5 - 3.2\lambda \end{vmatrix} = 0 \text{ ...... (1)}$$

$(18x10^5 - 9.6\lambda)(6x10^5 - 3.2\lambda) - (-6x10^5 - 1.6\lambda)(-6x10^5 - 1.6\lambda) = 0$

$\Rightarrow 1.08x10^{12} - 11.52x10^6\lambda + 3.72\lambda^2 - (3.6x10^{11} + 2.56\lambda^2 + 1.92x10^6\lambda) = 0$

$\Rightarrow 1.16\lambda^2 - 13.44x10^6\lambda + 7.2x10^{11} = 0$

By solving above equation we get the Eigen values are

$\lambda_1 = 11.53x10^6$

$\lambda_2 = 53.82x10^5$

The Natural frequencies are

$\omega_1 = \sqrt{\lambda_1} = \sqrt{11.53x10^6} = 3395.58$ rad/s

$\omega_2 = \sqrt{\lambda_2} = \sqrt{53.82x10^5} = 2319.9$ rad/s