PVP 23

Code: 23EC3603

III B.Tech - II Semester - Regular Examinations – APRIL 2026

DIGITAL SIGNAL PROCESSING
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	What is the frequency response of an LTI system?	L1	CO1
1.b)	State the condition for causality of an LTI system based on H(Z).	L1	CO2
1.c)	What is circular time shifting property of DFT?	L1	CO3
1.d)	Define DFT and IDFT.	L1	CO3
1.e)	What are the applications of FFT?	L1	CO3
1.f)	Write the advantages of FFT over DFT.	L1	CO3
1.g)	Compare Butterworth and Chebyshev filters.	L2	CO4
1.h)	Write the relationship between s-plane and z-plane poles in Impulse Invariant Transformation method.	L1	CO4
1.i)	What are symmetric and anti-symmetric impulse responses?	L1	CO4
1.j)	What is cascade realization?	L1	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	For the system described by $y(n) - 0.7y(n-1) + 0.12y(n-2) = x(n)$. Determine the frequency response and discuss on stability.	L3	CO1	5 M
	b)	Derive the conditions for causality and stability of LTI systems.	L3	CO2	5 M
OR					
3	Determine the impulse response and step response of a system described by the following equation. $y(n) = 0.6y(n-1)-0.08y(n-2)+x(n)$.		L3	CO2	10 M
UNIT-II					
4	Perform linear convolution of a sequence $x(n)=\{12,13,-1,-2,-3,4,5,6\}$ and $h(n)=\{2,1,-1\}$ using over-lap save method and verify the result with over-lap add method.		L3	CO3	10 M
OR					
5	a)	Compute the circular convolution of the sequences $x_1(n) = \{1,2,0,1\}$ and $x_2(n) = \{2,2,1,1,\}$.	L3	CO3	5 M
	b)	Prove any Three properties of DFT.	L3	CO3	5 M

UNIT-III					
6	a)	Draw Radix-2 DIF FFT algorithm and explain its characteristics.	L3	CO3	5 M
	b)	Determine the DFT of the sequence x(n) = (2,2,2,2,1,1,1,1) using Radix-2 DIT FFT algorithm.	L3	CO3	5 M
OR					
7	Compute IDFT of the sequence X(K) = {7, -0.707-j0.707, -j, 0.707-j0.707, 1, 0.707+j0.707, j, -0.707+j0.707} using Radix-2 DIF FFT algorithm.		L3	CO3	10 M
UNIT-IV					
8	Design a Butterworth IIR low pass filter with the following specifications: pass band gain k_1 = -1 dB, stop band attenuation k_2 = -40 dB, pass band edge frequency is 2 KHz, stop band edge frequency 10 KHz, Sampling frequency is 25 KHz. Use the bilinear transformation technique.		L5	CO4	10 M
OR					
9	Describe the IIR filter design using Bilinear Transformation method. Also sketch the s-plane to z-plane pole mapping.		L3	CO4	10 M
UNIT-V					
10	Differentiate between IIR and FIR filters. Discuss the various steps in designing FIR filter using window method.		L4	CO4	10 M

OR					
11	a)	Design an ideal LPF with the desired response $H_d(e^{j\omega}) = e^{-j5\omega}$; $\lvert\omega\rvert \leq \frac{\pi}{3}$ $= 0$; $\frac{\pi}{3} \leq \lvert\omega\rvert \leq \pi$ for N = 11 using Rectangular window.	L5	CO4	5 M
	b)	Realize a cascade and parallel realization for the system having difference equation y(n) + 0.1y(n-1) - 0.2y(n-2) = 3x(n) + 3.6x(n-1).	L3	CO5	5 M

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1) a) Definition – 2M

b) Condition for Causality – 2M

c) circular time shifting property – 2M

d) Definition of DFT – 1M

Definition of IDFT – 1M

e) Any two applications of FFT – 2M

f) Advantage of FFT over DFT – 2M

g) Any two Comparisons of Butterworth and Chebyshev filters – 2M

h) Relationship between s-plane & z-plane poles – 2M

i) Condition for symmetry – 1M

Condition of antisymmetric impulse response – 1M

j) Definition of Cascade realization – 2M

Part-B

2) a) Determination of H(z) — 2M

Poles of H(z) — 2M

Stability — 1M

b) Derivation of Causality Condition – 2M

Derivation of Stability Condition – 3M

3) Determination of $H(z)$ – 2M

Determination of $h(n)$ – 3M

Determination of Step response – 5M

4) Sub sequences in overlap-save Method – 2M

Finding linear Convolution – 3M

Sub sequences in overlap-add method – 2M

Finding linear Convolution using overlap-add method – 3M

5) a) Formula for circular Convolution – 2M

Determination of $y(n)$ – 3M

b) Any three properties – 5M

6) a) Radix-2 DIF FFT algorithm flowgraph – 3M

Characteristics – 2M

b) Radix-2 DIT FFT flowgraph – 2M

Computation of $X(K)$ – 3M

7) Radix-2 DIF FFT flow graph – 3M

Determination of $X^*(K)$ – 1M

Finding DFT of $X^*(K)$ – 3M

Conjugation of DFT of $X^*(K)$ – 2M

Dividing each o/p sample by N – 1M
and reordering $x(n)$.

8) Determination of Digital frequencies – 1M
Prewarping the specifications – 1M
order of the filter – 2M
Transfer function of normalized filter – 2M
Transfer function of analog filter – 2M
Transfer function of Digital filter – 2M

9) Design procedure of IIR filter using
Bilinear transformation method – 5M
s-plane to z-plane pole mapping – 5M

10) Any 5 differences between IIR & FIR filters – 5M
Design procedure of FIR filter using windows – 5M

11) a) Determination of hd(n) — 2M
Formula of Rectangular window – 1M
Determination of h(n) — 2M

b) System function H(z) – 1M
Cascade form realization – 2M
Parallel form realization – 2M

—— x ——

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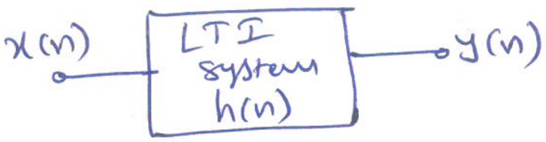
1) a)



$$h(n) \xleftrightarrow{DTFT} H(e^{j\omega})$$

$H(e^{j\omega})$ is called as frequency response of an LTI system and it is defined as

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-j\omega n}$$ — 2M

1) b) An LTI system with rational system function H(z) is causal if and only if ROC of H(z) is outside of an outermost pole including infinity. — 2M

or

An LTI system with rational system function H(z) is causal iff

i) ROC of H(z) is outside of an outermost pole

ii) Degree of the numerator polynomial of H(z) cannot be greater than the degree of the denominator polynomial. — 2M

1) c) Circular time shifting property of DFT is

$$x(n) \xleftrightarrow[N]{DFT} X(K)$$

then $$x(n-n_0) \xleftrightarrow[N]{DFT} W_N^{n_0 k} \cdot X(K)$$ — 2M

1) d) N-point DFT of a sequence x(n) is defined as

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk} \quad ; \quad K = 0, 1 \dots N-1$$

or

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nK/N} \quad ; K = 0, 1 \dots N-1$$ — 1M

Similarly Inverse DFT of X(K) is defined as

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \, W_N^{-nk} \quad ; \quad n = 0, 1 \dots N-1$$

or

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \, e^{j2\pi nk/N} \quad ; \quad n = 0, 1 \dots N-1$$

— 1M

1) e) Applications of FFT are

i) Linear filtering
ii) spectral analysis
iii) Correlation analysis

— 2M

1) f) The main advantage of FFT is to reduce the computational complexity of DFT, FFT algorithms are used. Therefore computation time of DFT can be reduced by using FFT.

— 2M

1) g)

Butterworth filter	Chebyshev filter
i) Poles of a Butterworth filter lie on a circle in s plane	i) Poles of Chebyshev filter lie on an ellipse in s-plane
ii) Transition band is more for Butterworth filters	ii) Transition band is less for Chebyshev filter
iii) order required is more than that of Chebyshev	iii) order of the filter is less than that of Butterworth.
iv) Butterworth filters are called as Maximally flat filters	iv) Chebyshev filters are called as equiripple filters

— Any two differences — 2M

1) h) Relationship between s-plane poles and z-plane poles in impulse invariant transformation method is

$$\boxed{z = e^{ST}}$$

where T is the sampling period.

— 2M

1) i) Condition for symmetric impulse response is

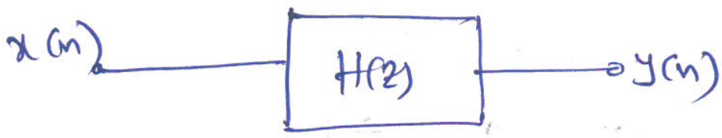
$$h(n) = h(N-1-n)$$ — 1M

where 'N' is the number of samples in h(n).

Condition for antisymmetric impulse response is

$$h(n) = -h(N-1-n)$$ — 1M

1) j) In cascade realization H(z) is decomposed into product of several simpler transfer functions.



When 'k' number of systems are connected in cascade with individual transfer functions $H_1(z), H_2(z) \ldots H_k(z)$, then it can be replaced by a single system with overall system function H(z) equivalent to product of individual functions.

$$\boxed{H(z) = H_1(z) \cdot H_2(z) \cdots H_K(z)}$$ — 2M

Part — B

2) a) Given $y(n) - 0.7y(n-1) + 0.12\,y(n-2) = x(n)$

applying z-transforms on both sides

$$Y(z) - 0.7z^{-1}Y(z) + 0.12z^{-2}Y(z) = X(z)$$

$$Y(z)\left[1 - 0.7z^{-1} + 0.12z^{-2}\right] = X(z)$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.7z^{-1} + 0.12z^{-2}}$$

Therefore frequency response of the system is

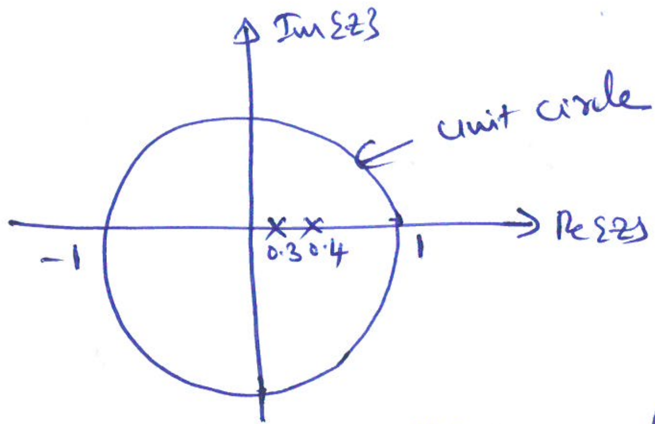
$$\boxed{H(z) = \frac{1}{1 - 0.7z^{-1} + 0.12z^{-2}}} \quad \text{— 2M}$$

factorizing the denominator polynomial of $H(z)$

$$H(z) = \frac{1}{(1-0.4z^{-1})(1-0.3z^{-1})} \quad \text{or}$$

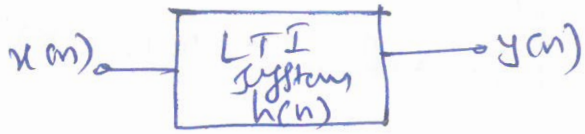
$$H(z) = \frac{z^2}{(z-0.4)(z-0.3)}$$

System has two poles at $z = 0.4$ and $z = 0.3$ — 2M



Since both the poles of $H(z)$ are located inside the unit circle. Therefore given system is a Stable system. — 1M

2) b) Condition for Causality :-



$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k) \cdot h(k)$$

$$y(n) = \sum_{k=-\infty}^{-1} x(n-k)\, h(k) + \sum_{k=0} x(n-k)\, h(k) + \sum_{k=1}^{\infty} x(n-k)\, h(k)$$

$$y(n) = \cdots + x(n+2)\, h(-2) + x(n+1)\, h(-1) + x(n)\, h(0)$$
$$+ x(n-1)\, h(1) + x(n-2)\, h(2) + \cdots$$

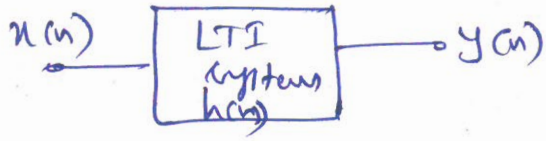
In the above equation o/p $y(n)$ is a function of present and past inputs iff $h(k) = 0$ when $k < 0$

$\therefore$ A DT LTI system is said to be causal iff

$$\boxed{h(n) = 0 \text{ when } n < 0}$$

— 2M

Condition for stability :-



$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k) \cdot h(k)$$

Let $x(n)$ be a bounded signal ie $|x(n)| < M_x < \infty$ for all 'n'.

then $|y(n)| = \left| \sum_{k=-\infty}^{\infty} x(n-k)\, h(k) \right|$

$$|y(n)| \le \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)|$$

$\because$ $x(n)$ is bounded, $\therefore$ $x(n-k)$ is also a bounded signal

$$|y(n)| \le \sum_{k=-\infty}^{\infty} |h(k)| \cdot M_x$$

$$|y(n)| \le M_x \cdot \sum_{k=-\infty}^{\infty} |h(k)|$$

In the above expression of $y(n)$ is bounded iff

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

$\therefore$ A DT LTI system is BIBO stable iff its impulse response $h(n)$ is absolutely summable ie

$$\boxed{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$$

— 3M

3) Given $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

$$y(n) - 0.6\,y(n-1) + 0.08\,y(n-2) = x(n)$$

applying z-transforms on both sides

$$Y(z) - 0.6 z^{-1} Y(z) + 0.08 z^{-2} Y(z) = X(z)$$

$$Y(z)\left[1 - 0.6 z^{-1} + 0.08 z^{-2}\right] = X(z)$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.6 z^{-1} + 0.08 z^{-2}}$$ — 2M

$$h(n) = IZT\{H(z)\}$$

$$H(z) = \frac{1}{(1 - 0.4 z^{-1})(1 - 0.2 z^{-1})}$$

using Partial fractions

$$H(z) = \frac{A}{1-0.2z^{-1}} + \frac{B}{1-0.4z^{-1}}$$

$$A = \left.\frac{1}{1-0.4z^{-1}}\right|_{z^{-1}=\frac{1}{0.2}=5} = \frac{1}{1-2} = -1$$

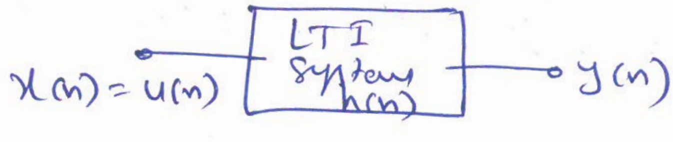
$$B = \left.\frac{1}{1-0.2z^{-1}}\right|_{z^{-1}=\frac{1}{0.4}=2.5} = \frac{1}{1-0.5} = 2$$

$$H(z) = \frac{-1}{1-0.2z^{-1}} + \frac{2}{1-0.4z^{-1}}$$

applying inverse z-transform on both sides, impulse response of the system is

$$\boxed{h(n) = -(0.2)^n u(n) + 2(0.4)^n u(n)} \quad \text{—— 3M}$$

Step Response of the system :-



$$y(n) = x(n) * h(n)$$

using Convolution property of z-transform

$$Y(z) = X(z) \cdot H(z)$$

$$\because \; x(n) = u(n) \; ; \quad X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{1}{(1-0.2z^{-1})(1-0.4z^{-1})(1-z^{-1})} \quad \text{—— 2M}$$

using Partial fractions

$$Y(z) = \frac{A}{1-0.2z^{-1}} + \frac{B}{1-0.4z^{-1}} + \frac{C}{1-z^{-1}}$$

$$A = \frac{1}{(1-0.4z^{-1})(1-z^{-1})}\Bigg|_{z^{-1}=\frac{1}{0.2}=5} = \frac{1}{(1-2)(1-5)} = \frac{1}{4}$$

$$B = \frac{1}{(1-0.2z^{-1})(1-z^{-1})}\Bigg|_{z^{-1}=\frac{1}{0.4}=2.5} = \frac{1}{(1-0.5)(1-2.5)} = \frac{-4}{3}$$

$$C = \frac{1}{(1-0.2z^{-1})(1-0.4z^{-1})}\Bigg|_{z^{-1}=1} = \frac{1}{(1-0.2)(1-0.4)} = 2.083$$

$$Y(z) = \frac{1/4}{1-0.2z^{-1}} + \frac{-4/3}{1-0.4z^{-1}} + \frac{2.083}{1-z^{-1}} \quad \text{— 2M}$$

applying inverse z-transforms, step response of the system is

$$\boxed{y(n) = \frac{1}{4}(0.2)^n u(n) - \frac{4}{3}(0.4)^n u(n) + 2.083\, u(n)} \quad \text{— 1M}$$

4) Given $x(n) = (12, 13, -1, -2, -3, 4, 5, 6)$

$h(n) = (2, 1, -1)$

Let $M=3$ and $L=3$; $N = L+M-1 = 3+3-1 = 5$

overlap-save Method :-

~~$x_1(n) = 12, 13, -1$~~

$x_1(n) = (0, 0, 12, 13, -1)$ $\qquad h(n) = (2, 1, -1, 0, 0)$

$x_2(n) = (13, -1, -2, -3, 4)$ $\qquad h(-n)_5 = (2, 0, 0, -1, 1)$

$x_3(n) = (-3, 4, 5, 6, 0)$

$x_4(n) = (6, 0, 0, 0, 0)$ $\qquad$ — 2M

$$y_1(n) = x_1(n) \circledast_5 h(n)$$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 12 \\ 13 \\ -1 \end{bmatrix} = \begin{bmatrix} -14 \\ 1 \\ 24 \\ 38 \\ -1 \end{bmatrix}$$

$$y_1(n) = (-14, 1, 24, 38, -1)$$

$$y_2(n) = x_2(n) \circledast_5 h(n)$$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 13 \\ -1 \\ -2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 33 \\ 7 \\ -18 \\ -7 \\ 7 \end{bmatrix}$$

$$y_2(n) = (33, 7, -18, -7, 7)$$

$$y_3(n) = x_3(n) \circledast_5 h(n)$$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} -3 \\ 4 \\ 5 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} -12 \\ 5 \\ 17 \\ 13 \\ 1 \end{bmatrix}$$

$$y_3(n) = (-12, 5, 17, 13, 1)$$

$$y_4(n) = x_4(n) \circledast_5 h(n)$$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 6 \\ -6 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore y_4(n) = (12, 6, -6, 0, 0$$

$y_1(n)$

$$\left[\ \cancel{-14}\ \ \cancel{1}\ \middle|\ 24\ \ 38\ \ -1\ \right]$$

$y_2(n)$

$$\left[\ \cancel{33}\ \ \cancel{7}\ \middle|\ -18\ \ -7\ \ 7\ \right]$$

$y_3(n)$

$$\left[\ \cancel{-12}\ \ \cancel{8}\ \middle|\ 17\ \ 13\ \ 1\ \right]$$

$y_4(n)$

$$\left[\ \cancel{12}\ \ \cancel{6}\ \middle|\ -6\ \ 0\ \ 0\ \right]$$

$$y(n) = (24 \quad 38 \quad -1 \quad -18 \quad -7 \quad 7 \quad 17 \quad 13 \quad 1 \quad -6)$$

— 3M

Overlap-add Method :-

$x_1(n) = (12, 13, -1, 0, 0)$ $\quad\therefore h(n) = (2, 1, -1, 0, 0)$

$x_2(n) = (-2, -3, 4, 0, 0)$ $\quad h(-n) = (2, 0, 0, -1, 1)$

$x_3(n) = (5, 6, 0, 0, 0)$

$y_1(n) = x_1(n) \circledast_5 h(n)$ — 2M

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 12 \\ 13 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 24 \\ 38 \\ -1 \\ -14 \\ 1 \end{bmatrix}$$

$y_1(n) = (24, 38, -1, -14, 1)$

$y_2(n) = x_2(n) \circledast_5 h(n)$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} -2 \\ -3 \\ 4 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \\ 7 \\ 7 \\ -4 \end{bmatrix}$$

$y_2(n) = (-4, -8, 7, 7, -4)$

$$\begin{bmatrix} 2 & 0 & 0 & -1 & 1 \\ 1 & 2 & 0 & 0 & -1 \\ -1 & 1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 17 \\ 1 \\ -6 \\ 0 \end{bmatrix}$$

$$y_3(n) = (10, 17, 1, -6, 0)$$

$y_1(n)$: $\boxed{24, \ 38, \ -1, \ \cancel{-14}, \ \cancel{1}}$

(M-1) Points add together

$y_2(n)$: $\boxed{\cancel{-4} \ \ \cancel{-8} \ \ 7 \ \ \cancel{7} \ \ \cancel{-4}}$

(M-1) Points add

$y_3(n)$: $\boxed{\cancel{10} \ \ \cancel{17} \ \ 1 \ \ -6 \ \ 0}$

$y(n)$: $\boxed{24 \ \ 38 \ \ -1 \ \ -18 \ \ -7 \ \ 7 \ \ 17 \ \ 13 \ \ 1 \ \ -6}$

$$\therefore y(n) = (24, 38, -1, -18, -7, 7, 17, 13, 1, -6)$$

— 3M

5) a) Given $x_1(n) = (1, 2, 0, 1)$ and $x_2(n) = (2, 2, 1, 1)$

Let $y(n) = x_1(n) \circledast_N x_2(n)$

$$y(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2((n-m))_N$$

Let $N = 4$

$$y(n) = \sum_{m=0}^{3} x_1(m) \cdot x_2((n-m))_4$$

— 2M

$x_1(m) = (1, 2, 0, 1) \qquad x_2(m) = (2, 2, 1, 1)$

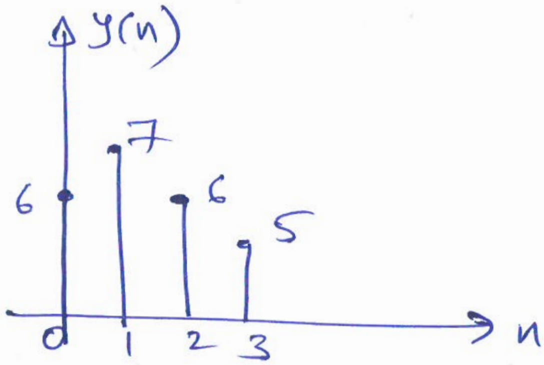
$x_2((-m))_4 = (2, 1, 1, 2)$

n	$x_1(m)$	$x_2((n-m))_4$	$y(n) = \sum_{m=0}^{3} x_1(m)\, x_2((n-m))_4$
0	$(1, 2, 0, 1)$	$(2, 1, 1, 2)$	$y(0) = (1\times2) + (2\times1) + (1\times2) = 6$
1	$(1, 2, 0, 1)$	$(2, 2, 1, 1)$	$y(1) = (1\times2) + (2\times2) + (1\times1) = 7$
2	$(1, 2, 0, 1)$	$(1, 2, 2, 1)$	$y(2) = (1\times1) + (2\times2) + (1\times1) = 6$
3	$(1, 2, 0, 1)$	$(1, 1, 2, 2)$	$y(3) = (1\times1) + (2\times1) + (1\times2) = 5$

$\therefore\ y(n) = (6, 7, 6, 5)$

Circular Convolution of $x_1(n)$ and $x_2(n)$ is

$\boxed{y(n) = 6, 7, 6, 5)}$



3M

5) b) i) Linearity property of DFT :-

Let $x_1(n) \xleftrightarrow[N]{DFT} X_1(K)$

$x_2(n) \xleftrightarrow[N]{DFT} X_2(K)$

then $a x_1(n) + b x_2(n) \xleftrightarrow[N]{DFT} a X_1(K) + b X_2(K)$

Proof

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n) W_N^{nK} \; ; \; K = 0, 1 \ldots N-1$$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n) W_N^{nK} \; ; \; K = 0, 1 \ldots N-1$$

$$DFT\{a x_1(n) + b x_2(n)\} = \sum_{n=0}^{N-1} [a x_1(n) + b x_2(n)] W_N^{nK}$$

$$DFT\{a x_1(n) + b x_2(n)\} = a\left[\sum_{n=0} x_1(n)\, W_N^{nk}\right]$$

$$+ b\left[\sum_{n=0}^{N-1} x_2(n)\, W_N^{nk}\right]$$

$$= a\, X_1(K) + b\, X_2(K)$$

$$\boxed{\therefore\ DFT\{a x_1(n) + b x_2(n)\} = a\, X_1(K) + b\, X_2(K)}$$

ii) Time reversal Property of DFT :-

Let $x(n) \xleftrightarrow[N]{DFT} X(K)$

then $x((-n))_N \xleftrightarrow[N]{DFT} X((-K))_N$

Proof.

$$DFT\{x(n)\} = X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

$$DFT\{x(-n)\} = \sum_{n=0}^{N-1} x(-n).\, W_N^{nk}$$

let $-n = l$

$$= \sum_{l=0}^{-(N-1)} x(l)\, W_N^{-lk}$$

$$= \sum_{n=0}^{N-1} x(n).\, W_N^{n(-k)}$$

Comparing above expression with definition of DFT

$$\boxed{DFT\{x(-n)\} = X(-k)}$$

$$\therefore\ x((-n))_N \longleftrightarrow X((-k))_N$$

iii) Time shifting Property of DFT :-

Let $x(n) \xleftrightarrow[N]{DFT} X(k)$

then $x(n-n_0) \xleftrightarrow[N]{DFT} W_N^{n_0 k}\, X(K)$

$$DFT\{x(n)\} = X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

$$DFT\{x((n-n_0)\} = \sum_{n=0}^{N-1} x(n-n_0)\, W_N^{nk}$$

let $n - n_0 = l$

$n = l + n_0$

$$= \sum_{l=-n_0}^{N-1-n_0} x(l)\, W_N^{(l+n_0)K}$$

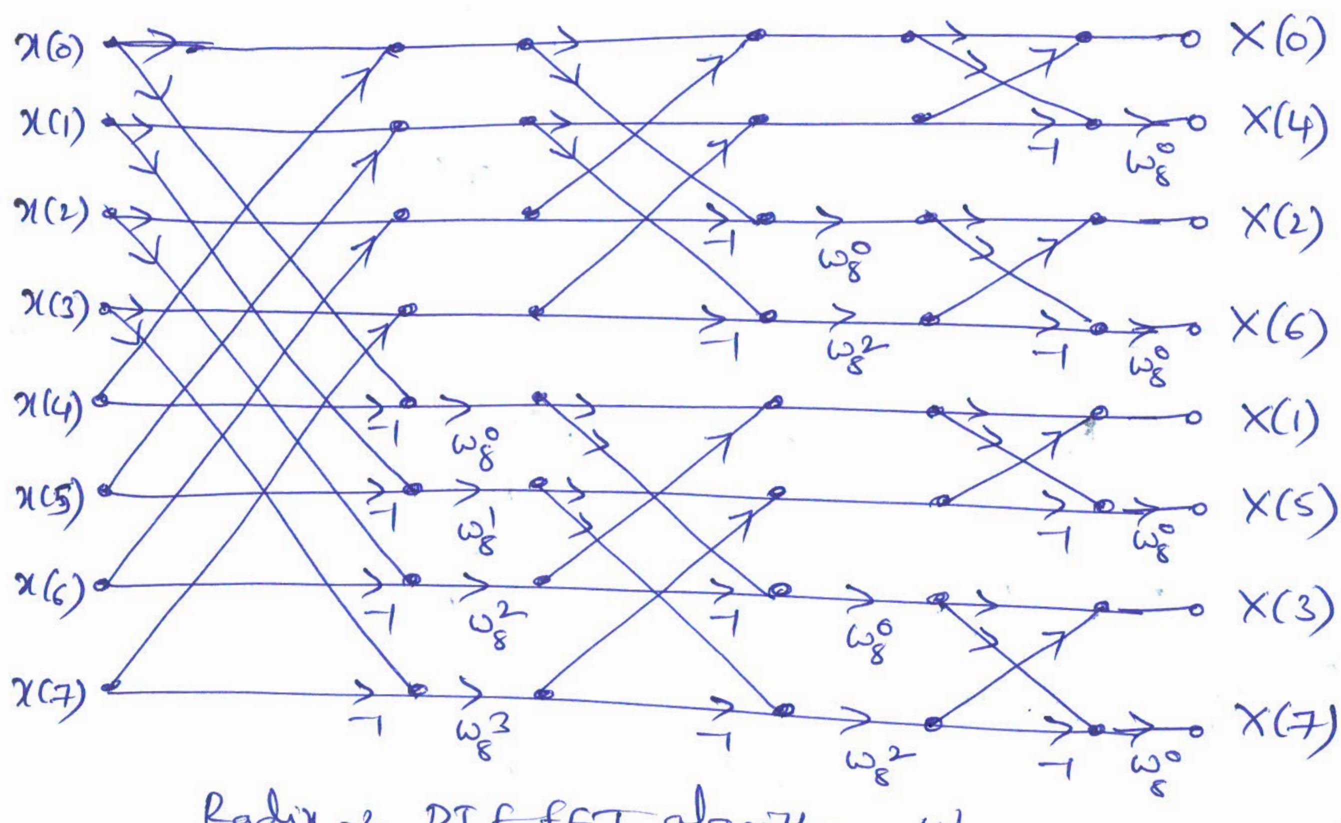
$$= \sum_{l=0}^{N-1} x(l)\, W_N^{lK} \cdot W_N^{n_0 k}$$

$$= W_N^{n_0 k} \left[\sum_{n=0}^{N-1} x(n)\, W_N^{nk} \right]$$

$$= W_N^{n_0 k} \cdot X(K)$$

$$\boxed{\therefore DFT\{x(n-n_0)\} = W_N^{n_0 K} \cdot X(K)}$$ — 5M

6) a) Radix-2 DIF FFT algorithm :-



Radix-2 DIF ~~f~~ FFT algorithm when N=8 — 3M

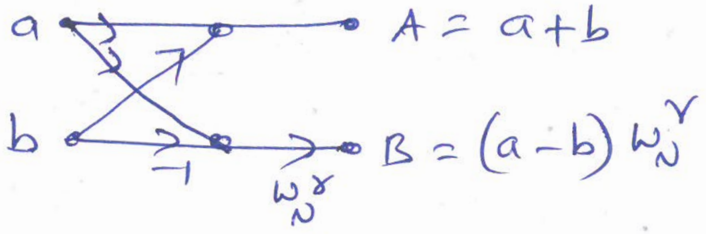
Characteristics of DIF FFT algorithm :-

i) I/p data is arranged in the normal order

ii) o/p data is arranged in the bit reversal order

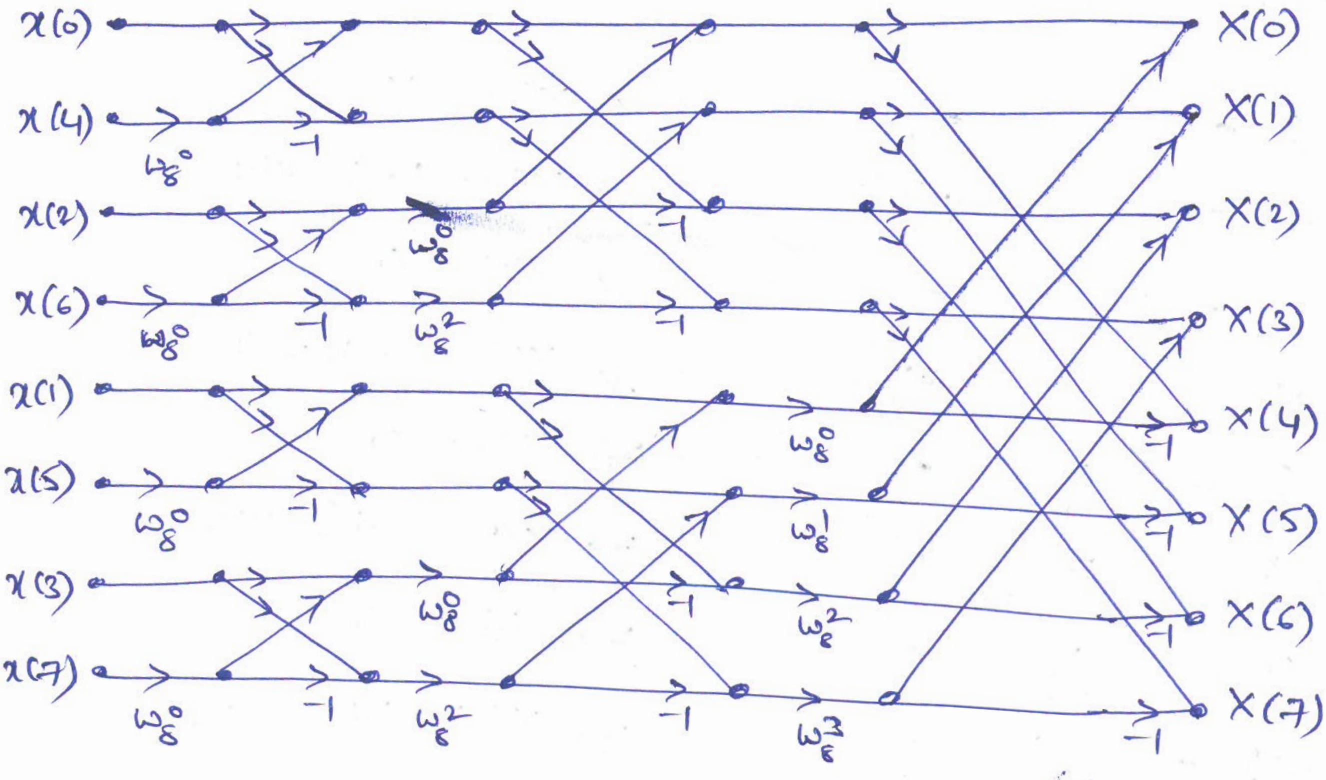
iii) Algorithm supports In-place calculations

iv) Basic computational block in the algorithm is called as "Butterfly"



— (2M)

6) b) Radix-2 DIT FFT algorithm :-



— (2M)

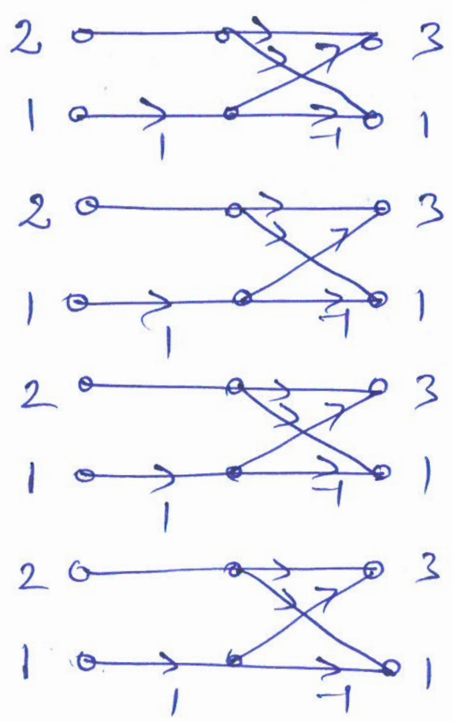
$$W_8^0 = e^{\frac{-j2\pi}{8}\cdot 0} = 1$$

$$W_8^1 = e^{\frac{-j2\pi}{8}\cdot 1} = 0.707 - j0.707$$

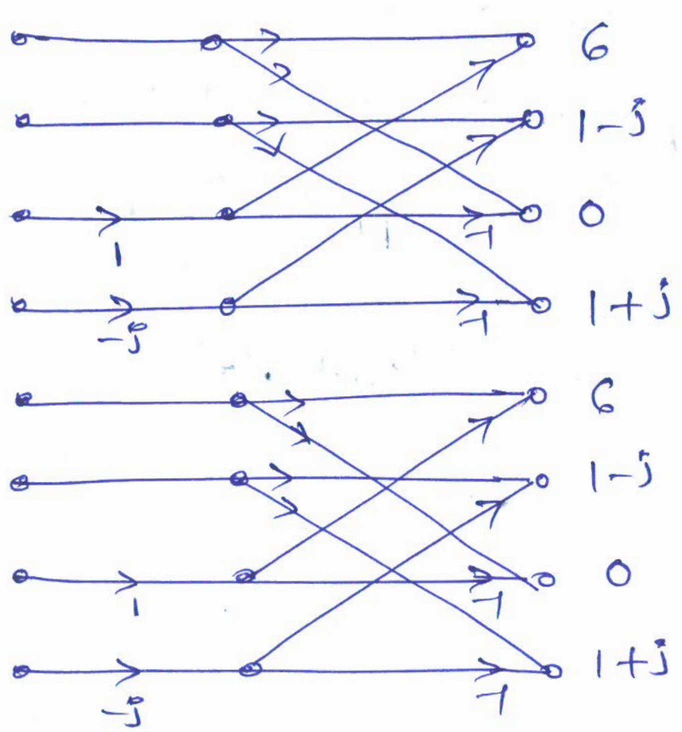
$$W_8^2 = e^{\frac{-j2\pi}{8}\cdot 2} = e^{\frac{-j\pi}{2}} = -j$$

$$W_8^3 = e^{\frac{-j2\pi}{8}\cdot 3} = e^{-j3\pi/4} = -0.707 - j0.707$$

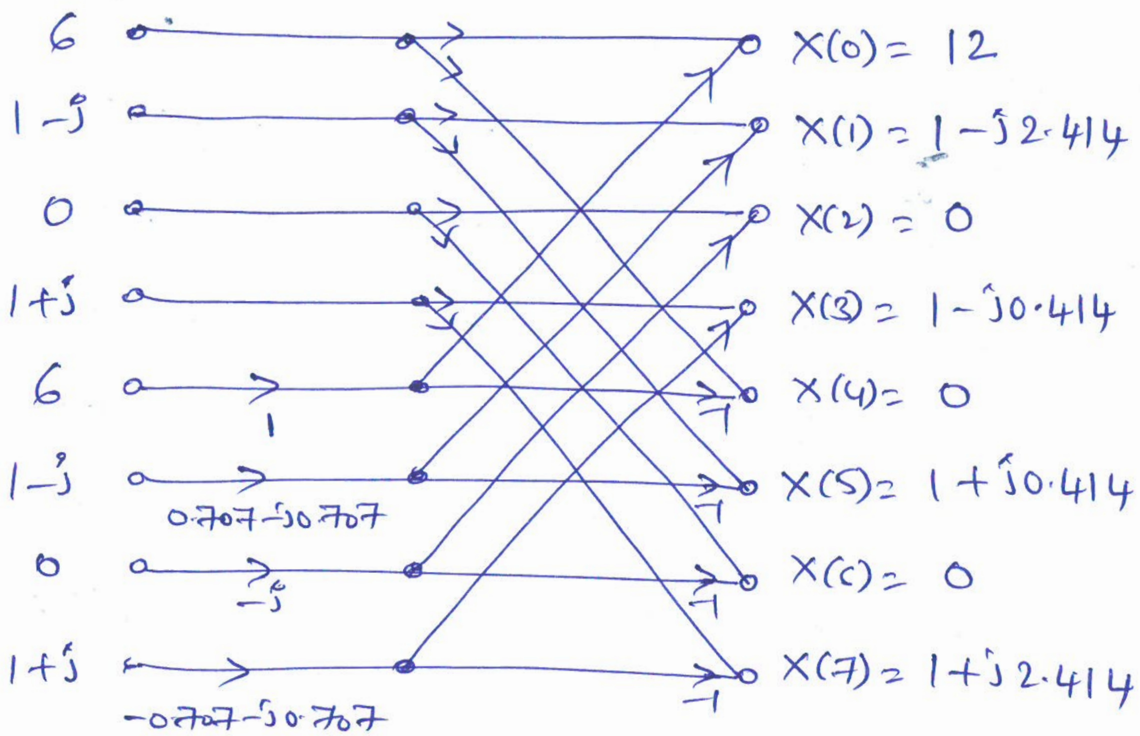
First stage :-



Second stage :-



Third stage :-

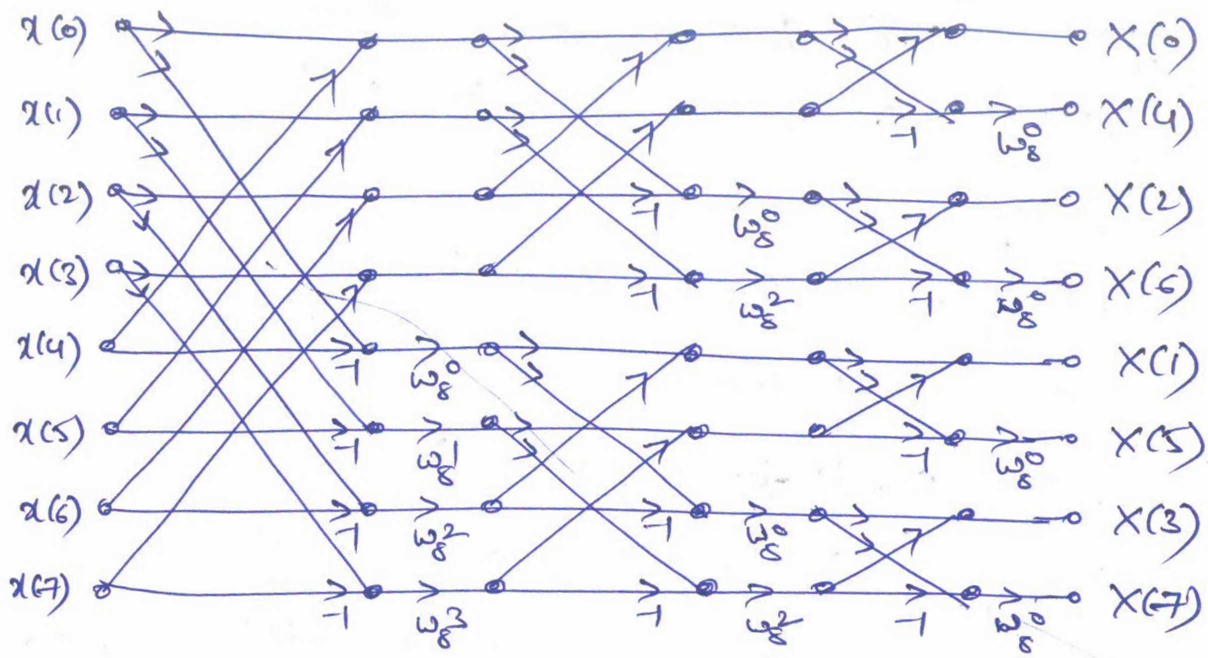


∴ DFT of the given sequence is

$X(k) = (12, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414)$

— (3M)

7) Radix-2 DIF FFT algorithm :-

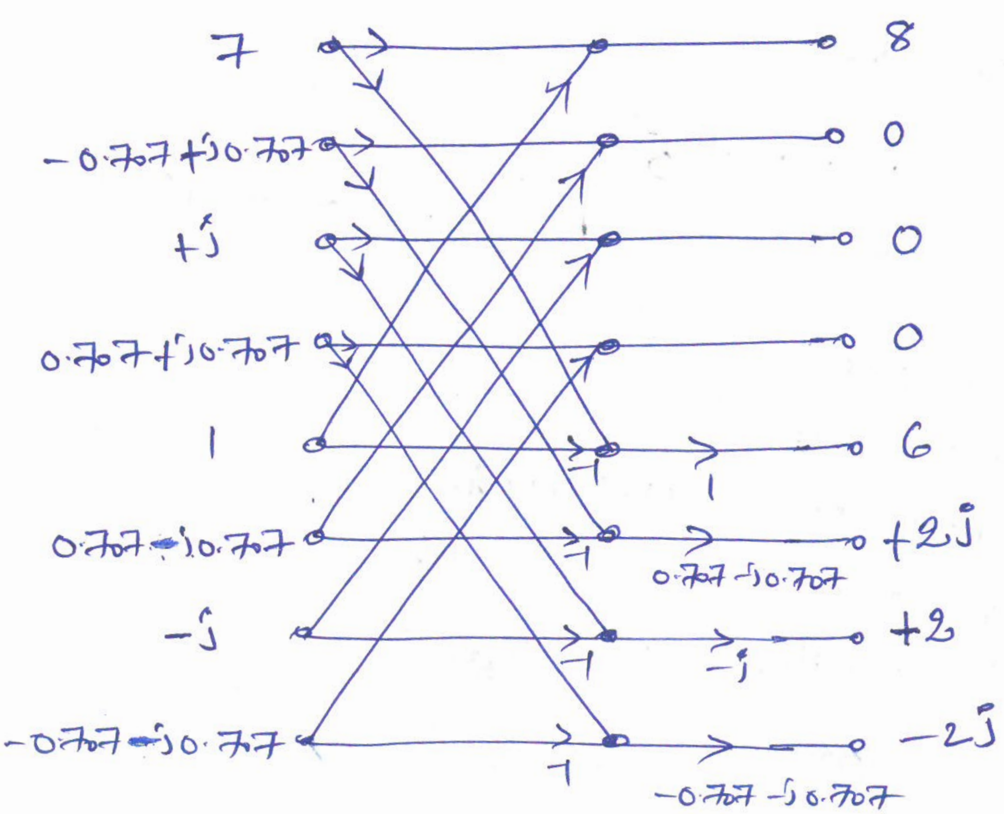


$W_8^0 = 1$; $W_8^1 = 0.707 - j0.707$ — 3M

$W_8^2 = -j$; $W_8^3 = -0.707 - j0.707$

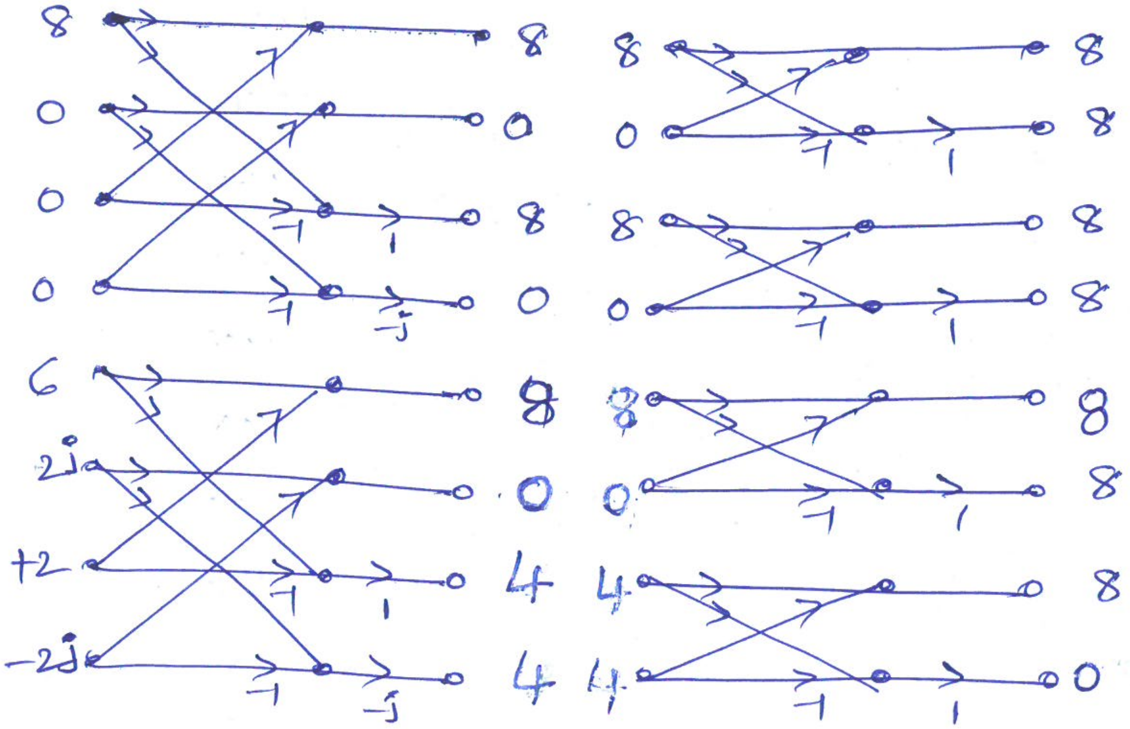
Step 1: obtain $X^*(k)$; Step 2: Determine DFT of $X^*(k)$ using FFT algorithm

First stage :-



Second stage

Third stage



— (4M)

$\therefore x^*(k) = (8, 8, 8, 8, 8, 8, 8, 0)$

$X(k) = (8, 8, 8, 8, 8, 8, 8, 0)$

Dividing each o/p sample by 8; IDFT of the given function is

$$\boxed{x(n) = (1, 1, 1, 1, 1, 1, 1, 0)}$$

— (3M)

8) Given $k_1 = -1 \text{ dB}$

$k_2 = -40 \text{ dB}$

$\Omega_u = 2\pi(2000) = 4000\pi$ rad/sec

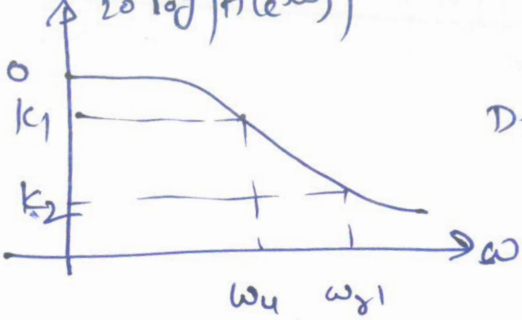
$\Omega_r' = 2\pi(10{,}000) = 20{,}000\pi$ rad/sec

$F = 25$ KHz ; $T = \frac{1}{25000}$

$\omega_u = \Omega_u \cdot T = \frac{4000\pi}{25000} = 0.16\pi$ rad

$\omega_r' = \Omega_r' \cdot T = \frac{20000\pi}{25000} = 0.8\pi$ rad

— (1M)



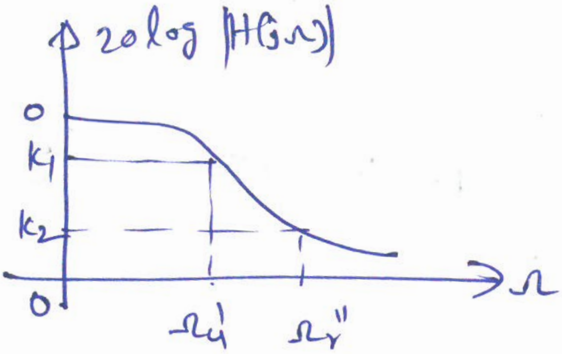
Digital LPF : H(z)

Step1 : Prewarping with $T = 1$ sec

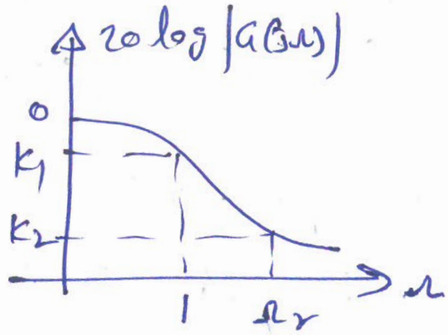
$$\Omega_u' = 2\tan\left(\frac{\omega_u}{2}\right) = 2\tan(0.08\pi) = \cancel{0.0088}\ 0.513 \text{ rad/}$$

$$\Omega_r'' = 2\tan\left(\frac{\omega_r'}{2}\right) = 2\tan(0.4\pi) = 6.155 \text{ rad/sec} \quad \text{— (1)}$$

Step 2 : Design analog filter with prewarped specifications



Analog LPF : H(s)



Prototype LPF : G(s)

$$\Omega_r = \frac{\Omega_r''}{\Omega_u'} = \frac{6.155}{0.513} = 11.998$$

$$n = \frac{\log\left[\left(10^{-k_1/10}-1\right)/\left(10^{-k_2/10}-1\right)\right]}{2\log\left[1/\Omega_r\right]}$$

$$n = \frac{\log\left[\left(10^{0.1}-1\right)/\left(10^{4}-1\right)\right]}{2\log\left[1/11.998\right]} = 2.126$$

— 2M

take $n = 3$

Transfer function of 3rd order normalized LPF is

$$H_n(s) = \frac{1}{s+1}\prod_{k=1}^{n-1/2}\frac{1}{s^2+b_k s+1}$$

where $b_k = 2\sin\left[\frac{(2k-1)\pi}{2n}\right] = 2\sin\left[\frac{(2-1)\pi}{6}\right] = 1$

$$\therefore H_3(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$\because K_1 \neq -3\,dB$

Cutoff frequency of prototype LPF is

$$\Omega_c = \frac{1}{(10^{-K_1/10}-1)^{1/2n}} = \frac{1}{(10^{0.1}-1)^{1/6}} = 1.252$$ — 2M

Transfer function of Prototype LPF is obtained by applying LP → LP transformation is

$$G(s) = H_3(s)\Big|_{s=\frac{s}{\Omega_c}} = \frac{s}{1.252}$$

Transfer function of analog LPF is obtained by applying LP → LP transformation ie

$$H(s) = G(s)\Big|_{s=\frac{s}{\Omega_u'}} = \frac{s}{0.513}$$

$$\therefore H(s) = H_3(s)\Big|_{s=\frac{s}{(1.252)(0.513)}} = \frac{s}{0.642}$$

$$H(s) = \frac{1}{\left[\frac{s}{0.642}+1\right]\left[\left(\frac{s}{0.642}\right)^2+\frac{s}{0.642}+1\right]}$$

$$H(s) = \frac{0.265}{(s+0.642)(s^2+0.642s+0.412)}$$ — 2M

Step 3: Transfer function of Digital LPF is obtained by applying Bilinear transformation ~~ie~~ with $T = 1\,sec$ ie

$$H(z) = H(s)\Big|_{s = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{0.265}{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.642\right]\left[4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^2+1.284\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.4\right.}$$

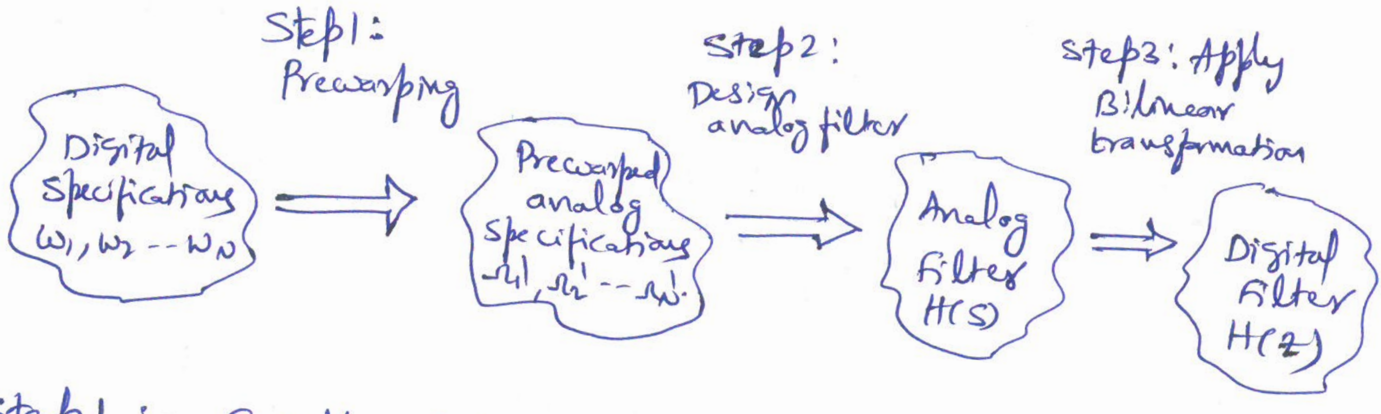
$$H(z) = \frac{0.265\,(1+z^{-1})^3}{\left[2.642-1.358z^{-1}\right]\left[5.696-7.176\,z^{-1}+3.128\,z^{-2}\right]}$$

or

$$\boxed{H(z) = \frac{0.0176(1+z^{-1})^3}{\left[1-0.514z^{-1}\right]\left[1-1.26z^{-1}+0.549\,z^{-2}\right]}}$$

— 2M

9) IIR filter Design using Bilinear transformation method :-



Step1 :- For the given specifications, obtain the prewarped analog specifications using the formula

$$\Omega_i^* = \frac{2}{T}\tan\left(\frac{\omega_i}{2}\right) \qquad \text{Choose } T = 1\,\text{sec}$$

$$\therefore \Omega_i^* = 2\tan\left(\frac{\omega_i}{2}\right)$$

Step 2: Design Analog filter using Prewarped analog specifications

Step 3: Apply Bilinear transformation method with T = 1 sec to obtain the system function of the digital filter ie

$$H(z) = H(s)\Big|_{s = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} \quad \text{— SM}$$

Relationship between s-plane poles and z-plane Poles :-

Relationship between s-plane and z-plane in Bilinear transformation method is

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

or

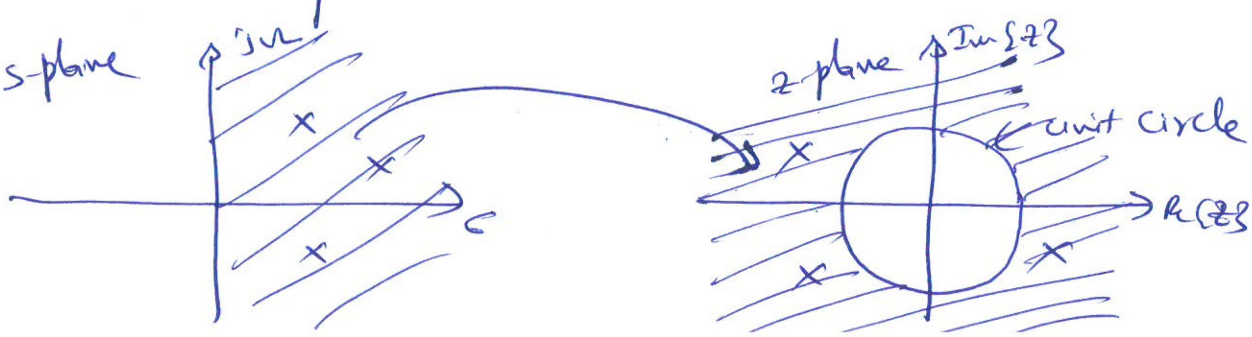
$$s = \frac{2}{T}\left[\frac{z-1}{z+1}\right] \qquad \text{let } z = re^{j\omega}$$

$$s = \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right] = \frac{2}{T}\left[\frac{r\cos\omega - 1 + jr\sin\omega}{r\cos\omega + 1 + jr\sin\omega}\right]$$

$$s = \frac{2}{T}\left[\frac{r^2 - 1 + j2r\sin\omega}{r^2 + 1 + 2r\cos\omega}\right] \qquad \text{let } s = \sigma + j\Omega$$

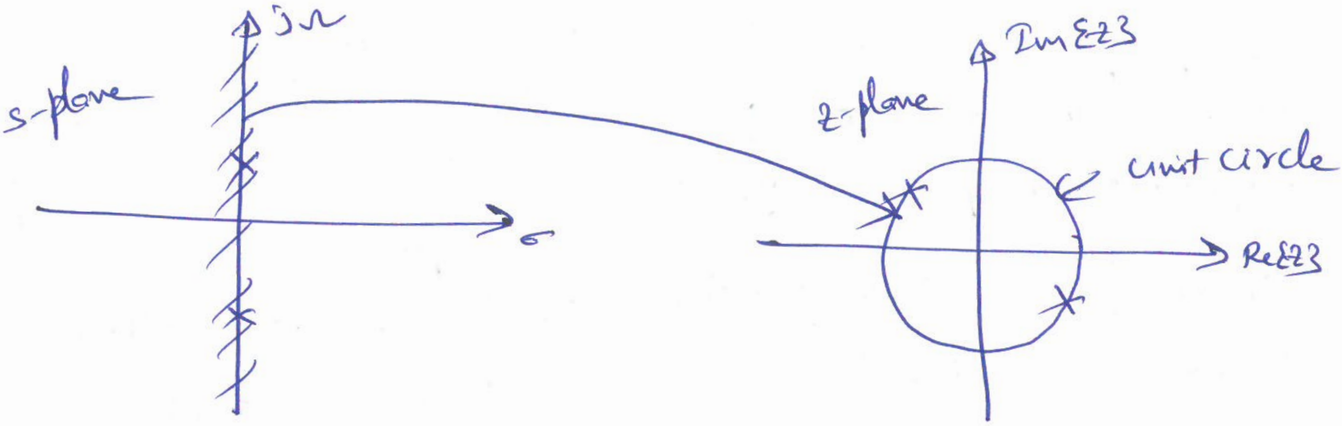
$$\therefore \quad \sigma = \frac{2}{T}\left[\frac{r^2-1}{r^2+1+2r\cos\omega}\right]$$

i) In the above expression when $\sigma > 0$; $r > 1$



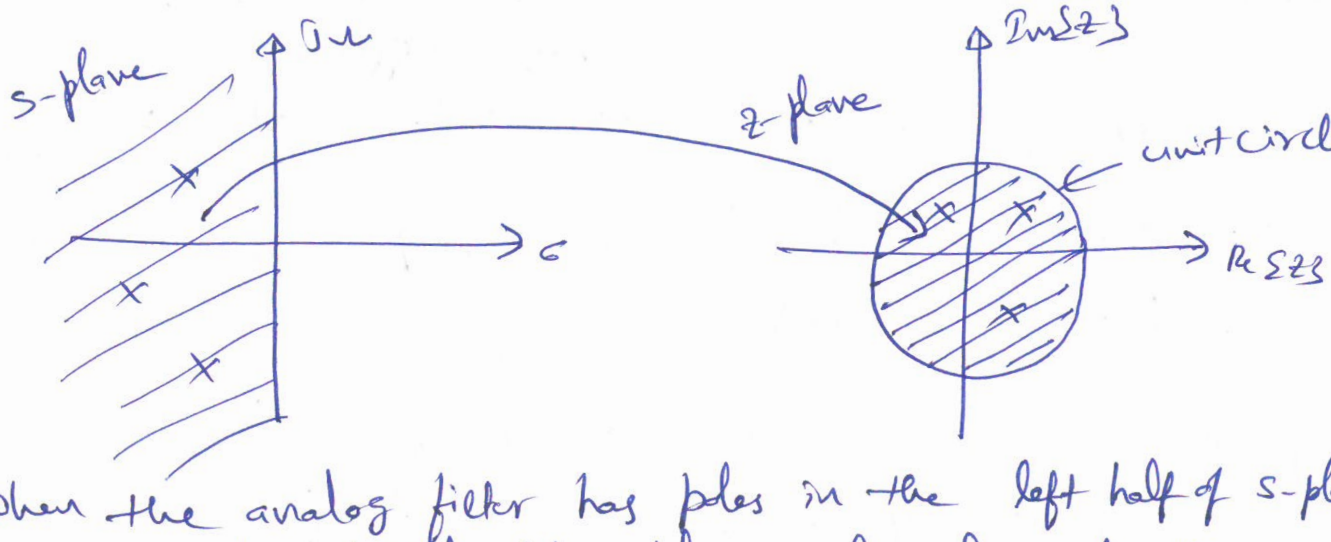
ie when the analog filter has poles in the right half of s-plane, the corresponding digital filter poles are located outside the unit circle in z-plane. Therefore the right half of s-plane is mapped to outside of the unit circle in z-plane.

ii) when $\sigma = 0$ then $r = 1$ ie $|z| = 1$



when the analog filter has poles on the imaginary axis of s-plane, then the corresponding digital filter poles are located on the unit circle. Therefore ~~when the analog~~ imaginary axis of s-plane is mapped on to the unit circle in z-plane.

iii) when $\sigma < 0$; then $r < 1$ ie $|z| < 1$



When the analog filter has poles in the left half of s-plane, the corresponding digital filter poles are located inside the unit circle. Therefor the LHP of s-plane is mapped to interior of unit circle in z-plane.

— (SM)

10) Differences between IIR and FIR filters.

IIR filters	FIR filters
1. Impulse Response of IIR filter consists of infinite number of samples	1. Impulse Response of FIR filter consists of finite number of samples.
2. The Specifications include the desired characteristics for magnitude response only.	2. The specifications include the desired characteristics for both magnitude and phase response.
3. Linear phase characteristics cannot be achieved	3. Linear phase filters can be easily designed
4. IIR filters are not always stable	4. FIR filters are always stable
5. High selectivity can be achieved with lower order transfer functions	5. High selectivity can be achieved by using higher order for the transfer function
6. Errors due to round off noise are more	6. Errors due to roundoff noise are less.

— Any 5 differences — 5M

Design steps of FIR filter using window Method :-

Step 1: Choose the ideal frequency response of the desired filter ie $H_d(e^{j\omega})$

Step 2: Determine Inverse DTFT of $H_d(e^{j\omega})$ to obtain the impulse response of the desired filter ie $h_d(n)$.

Step 3: Choose the window type from the table in suchaway that the stopband attenuation of selected window must exceed K_2.

Step 4: Choose the number of samples in a window using

$$\omega_2 - \omega_1 \geq \frac{K.2\pi}{N}$$

ie $N \geq \frac{K.2\pi}{\omega_2 - \omega_1}$ where K is an integer

Step 5: Impulse Response of FIR filter is obtained by multiplying infinite duration impulse response $h_d(n)$ with a finite duration window function $\omega(n)$. ie

$$h(n) = h_d(n) \cdot \omega(n)$$

Step 6: for a LPF ; $h(n) = \frac{\sin \omega_c (n-\alpha)}{\pi(n-\alpha)} \cdot \omega(n)$

Choose $\omega_c = \omega_1$ and

$$\alpha = \frac{N-1}{2}$$

Step 7: Therefore impulse response of FIR LPF is

$$h(n) = \frac{\sin \omega_c \left(n - \frac{N-1}{2}\right)}{\pi \left(n - \frac{N-1}{2}\right)} \cdot \omega(n) \quad ; \quad 0 \leq n \leq N-1$$

$$= 0 \quad ; \text{ otherwise.}$$

— SM

11) a) Given

$$H_d(e^{j\omega}) = e^{-j5\omega} \quad ; \; |\omega| \leq \frac{\pi}{3}$$

$$= 0 \quad ; \; \frac{\pi}{3} < |\omega| < \pi$$

N = 11

Rectangular window

Step 1: $h_d(n) = IDTFT\{H_d(e^{j\omega})\}$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\cdot e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{-j5\omega}\cdot e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{-\pi/3}^{\pi/3} e^{j(n-5)\omega}\, d\omega$$

$$h_d(n) = \frac{\sin\frac{\pi}{3}(n-5)}{\pi(n-5)}$$ — 2M

Step 2: Given Rectangular window. $W_R(n) = 1 ; 0 \le n \le 10$
$= 0 ;$ otherwise

Step 3: Given $N = 11$ — 1M

Step 4: Impulse response of FIR filter is obtained by

$$h(n) = h_d(n)\cdot W_R(n)$$

where $W_R(n)$ is a rectangular window function

$$W_R(n) = 1 \quad ; \quad 0 \le n \le 10$$
$$= 0 \quad ; \quad \text{otherwise}$$

Step 5: Choose $\alpha = \frac{N-1}{2} = \frac{11-1}{2} = 5$

Step 6: $$h(n) = \frac{\sin\frac{\pi}{3}(n-5)}{\pi(n-5)}\cdot W_R(n)$$

$$\boxed{\begin{aligned} h(n) &= \frac{\sin\frac{\pi}{3}(n-5)}{\pi(n-5)} \quad ; \quad 0 \le n \le 10 \\ &= 0 \quad ; \quad \text{otherwise} \end{aligned}}$$

— 2M

11) b) Given

$$y(n) + 0.1\,y(n-1) - 0.2\,y(n-2) = 3x(n) + 3.6\,x(n-1)$$

applying z-transforms on both sides

$$Y(z) + 0.1\,z^{-1}Y(z) - 0.2\,z^{-2}Y(z) = 3X(z) + 3.6\,z^{-1}X(z)$$

$$Y(z)\left[1 + 0.1\,z^{-1} - 0.2\,z^{-2}\right] = X(z)\left[3 + 3.6\,z^{-1}\right]$$

$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6\,z^{-1}}{1 + 0.1\,z^{-1} - 0.2\,z^{-2}} \quad \text{—— 1 M}$$

Cascade form realization :-

$$H(z) = H_1(z) \cdot H_2(z)$$

$$H(z) = \frac{3 + 3.6\,z^{-1}}{(1 + 0.5\,z^{-1})(1 - 0.4\,z^{-1})}$$

$$H(z) = \left(\frac{3 + 3.6\,z^{-1}}{1 + 0.5\,z^{-1}}\right)\left(\frac{1}{1 - 0.4\,z^{-1}}\right)$$

Coefficients of $H_1(z)$ are

$b_0 = 3$

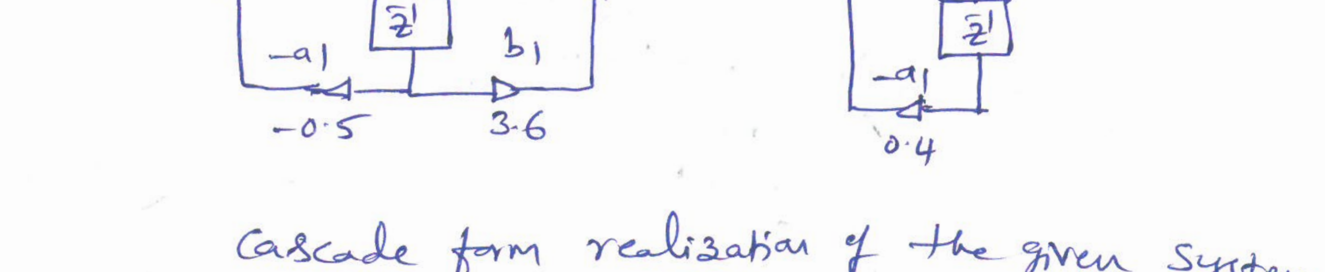
$b_1 = 3.6$

$a_1 = 0.5$

Coefficients of $H_2(z)$ are

$b_0 = 1$

$a_1 = -0.4$



Cascade form realization of the given system

—— 2M

Parallel form realization :- $H(z) = H_1(z) + H_2(z)$

$$H(z) = \frac{3 + 3.6 z^{-1}}{(1+0.5 z^{-1})(1-0.4 z^{-1})} = \frac{A}{1+0.5 z^{-1}} + \frac{B}{1-0.4 z^{-1}}$$

$$A = \left.\frac{3 + 3.6 z^{-1}}{1 - 0.4 z^{-1}}\right|_{z^{-1} = -2} = \frac{3 - 7.2}{1 + 0.8} = -2.33$$

$$B = \left.\frac{3 + 3.6 z^{-1}}{1 + 0.5 z^{-1}}\right|_{z^{-1} = 2.5} = \frac{3 + 9}{1 + 1.25} = 5.33$$

$$\therefore H(z) = \frac{-2.33}{1+0.5 z^{-1}} + \frac{5.33}{1 - 0.4 z^{-1}}$$

$$H(z) = H_1(z) + H_2(z)$$

Coefficients of $H_1(z)$ are

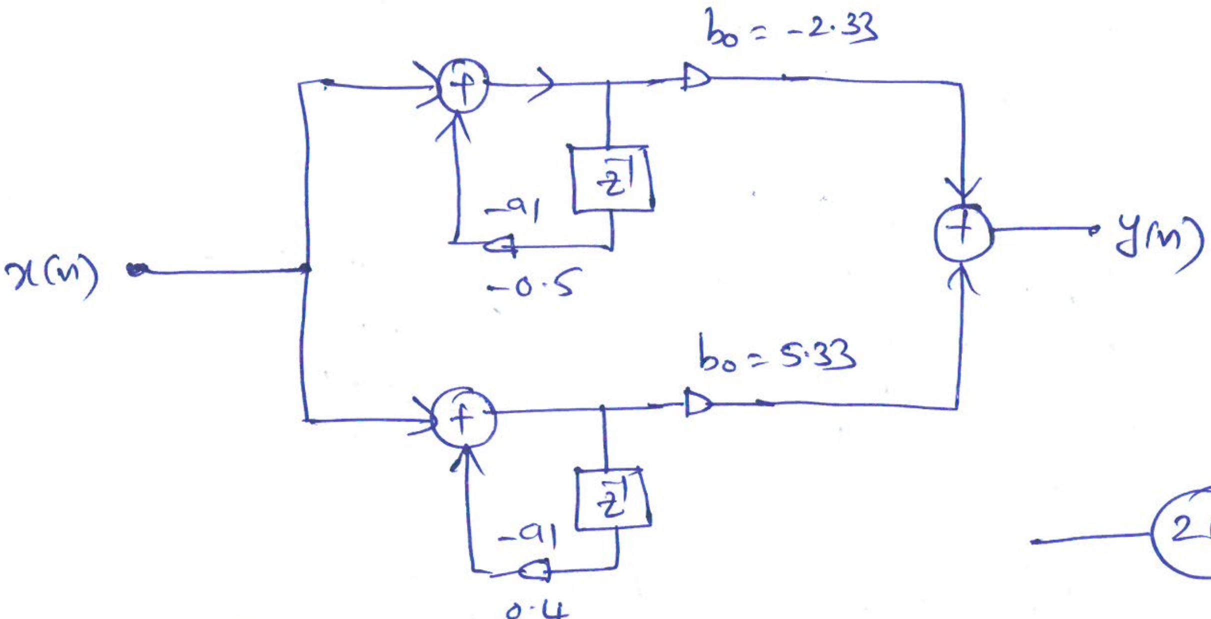
$b_0 = -2.33$

$a_1 = 0.5$

Coefficients of $H_2(z)$ are

$b_0 = 5.33$

$a_1 = -0.4$



2M

Parallel form realization of the given system

—— X ——