		OR			
9	a)	Evaluate the Laplace transform of $\frac{\cos at-\cos bt}{t}$	L3	CO3	5 M
	b)	Evaluate the inverse Laplace transform of $log\left(\frac{s+1}{s-1}\right)$	L3	CO3	5 M
		UNIT-V			
10		Express $f(x)=\|x\|, -\pi<x<\pi$ in a Fourier series. Hence deduce $\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots$	L3	CO5	10 M
		OR			
11		Find the Fourier transform of $f(x)=\begin{cases} a-\|x\|, & \|x\|\le a \\ 0, & \|x\|>a \end{cases}$ Hence show that $\int_0^\infty \frac{\sin^2 x}{x^2}dx=\frac{\pi}{2}$	L4	CO5	10 M

Code: 23BS1303

II B.Tech - I Semester – Regular / Supplementary Examinations
NOVEMBER 2025

NUMERICAL METHODS AND TRANSFORM TECHNIQUES
(MECHANICAL ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1. a)	Explain Bisection method.	L2	CO1
b)	Obtain an iterative formula to find the root of $\sqrt[3]{N}$ by Newton Raphson Method.	L3	CO1
c)	Find the first derivative of Newton's forward interpolation formula at $x=x_0$	L1	CO2
d)	Evaluate $\int_0^3 \frac{dx}{1+x^2}$ by trapezoidal Rule.	L3	CO2
e)	State the Taylor's series method formula.	L1	CO2
f)	Solve the differential equation with the given conditions by Picard's method. $\frac{dy}{dx}=1+xy$, $y_0=1$ when $x_0=0$ up to y_2.	L3	CO2
g)	State and prove the first shifting property.	L3	CO3
h)	Find L$\{t^2e^{2t}\}$	L1	CO3
i)	Explain Dirichlet's conditions.	L2	CO5
j)	Find a_0 if $f(x)=x^2$ in the $interval$ $(0,2\pi)$ using Fourier series.	L1	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Determine a positive root of $x^3 - 4x + 1 = 0$, correct up to three decimal places by using Regula-falsi method.	L3	CO2	5 M
	b)	Obtain the missing terms in the following table. X: 1, 2, 3, 4, 5, 6, 7, 8 Y: 2, 4, 8, -, 32, -, 128, 256	L4	CO2	5 M
		OR			
3		From the following table estimate the number of students who obtained marks between 40 and 45 using Newton's forward interpolation formula. Marks: 30-40, 40-50, 50-60, 60-70, 70-80 Number of students: 31, 42, 51, 35, 31	L3	CO4	10 M
		UNIT-II			
4		Find $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ at x=1.1 and at x=1.6 for the following table given below. X: 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6 Y: 7.989, 8.403, 8.781, 9.129, 9.451, 9.750, 10.031	L3	CO2	10 M
		OR			
5	a)	A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given as follows:	L3	CO2	5 M

t(s)	0	10	20	30	40	50	60	70	80
a(m/s²)	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

By $Simpson's\ \frac{1}{3}rd$ rule, find the velocity at t=80 sec.

			BL	CO	Max. Marks
	b)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's Rule.	L4	CO4	5 M
		UNIT-III			
6		Apply Milne's method to find the solution of the differential quation $y' = x - y^2$ *in the range* $0 \le x \le 1$ for the condition y(0)=0, h = 0.2.	L3	CO2	10 M
		OR			
7	a)	Apply the Runge-Kutta method of fourth order to find an approximate value of y at x=0.1 if $\frac{dy}{dx} = x + y^2$, given that y=1 when x=0 in steps of h=0.1	L3	CO2	5 M
	b)	Using modified Euler's method to find the value of y satisfying the equation $\frac{dy}{dx} = \log(x+y)$ for x=1.2, correct up to four decimal places by taking h=0.2. Given that y(1)=2.	L4	CO4	5 M
		UNIT-IV			
8	a)	Determine the inverse Laplace transform of $\left\{\frac{4s+15}{16s^2-25}\right\}$ using partial fractions.	L3	CO3	5 M
	b)	Evaluate $L\{te^t sint\}$	L4	CO3	5 M

II nd B. Tech – I Semester – Regular NOV 2025 (PVP-23)

Numerical Methods and Transform techniques (23BS1303)

(Mechanical Engineering)

PART – A

1. (a) An eq. of the form $f(x)=0$ has ~~exactly~~ at least one root b/w two real nos x_0, x_1. $f(x_0)$ & $f(x_1)$ will have opposite signs.

Ist approximation $x_2 = \frac{x_1+x_2}{2}$ —— (1m)

If $f(x_2)=0$ then x_2 is the root.

If $\left.\begin{matrix} f(x_1)<0 \\ f(x_2)>0 \end{matrix}\right\}$ then the root lies b/w x_1 and x_2 and vice verse. Each time continue the same procedure till the desired degree of accuracy. —— (1m)

(b) let $x = N^{1/3}$

$\Rightarrow x^3 - N = 0$

let $f(x) = x^3 - N$; $f'(x) = 3x^2$

$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ —— (1m)

$= x_n - \frac{(x_n^3 - N)}{3x_n^2}$

$= \frac{2x_n^3 + N}{3x_n^2}$ —— (1m)

(c) $\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \cdots\cdots\right]$ → (2m)

$\int^{x_n} f(x)dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$ → (1m)

(2m)

x	0	1	2	3
y	1	0.5	0.2	0.1

$\therefore \int_0^3 \frac{dx}{1+x^2} = \frac{1}{2}\left[1.1 + 2(0.7)\right]$

$= 1.25$ → (1m)

(e) $y(x) = y(x_0) + \frac{(x-x_0)}{\lfloor 1} y'(x_0) + \frac{(x-x_0)^2}{\lfloor 2} y''(x_0) + \cdots \longrightarrow$ (2m)

(or)

put $x - x_0 = h$.

$y(x) = y_0 + h \cdot y_0' + \frac{h^2}{\lfloor 2} y_0'' + \frac{h^3}{\lfloor 3} y_0''' + \cdots$

(f) $\frac{dy}{dx} = 1 + xy$; $x_0 = 0$; $y_0 = 1$

$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$

$= 1 + \int_0^x (1+0)\, dx = 1 + x$ —— (1m)

$y_2 = y_0 + \int_0^x f(x, y_1)\, dx$

$= 1 + \int_0^x [1 + x(1+x)]\, dx$.

$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3}$ —— (1m)

(g) $L\{e^{at} f(t)\} = \int_0^\infty e^{-st} f(t)\, dt \Rightarrow \int_0^\infty e^{-st} \cdot e^{at}\, dt = \int_0^\infty e^{-(s-a)t}\, dt$ —— (1m)

$= \int_0^\infty e^{-ut} f(t)\, dt$ where $u = s - a$

$= F(u)$

$= F(s-a)$. —— (1m)

(h) $L\{t^2 e^{2t}\} = ?$

$L\{e^{2t}\} = \frac{1}{s-2} \longrightarrow$ (1m)

$L\{t^2 e^{2t}\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s-2}\right)$

$= \frac{2}{(s-2)^3} \longrightarrow$ (1m)

(or) $L\{t^2\} = \frac{\lfloor 2}{s^3} \longrightarrow$ (1m)

$L\{e^{2t} t^2\} = \frac{2}{(s-2)^3} \longrightarrow$ (1m)

[By applying 1st shifting Th]

(i) $f(x)$ is well defined, single-valued, $f(x)$ has a finite no. of finite discontinuities in the interval. $f(x)$ has at most a finite no. of maxima and minima in the interval. $\longrightarrow$ (2m)

(j) $a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x)\, dx = \frac{1}{\pi} \int_0^{2\pi} x^2\, dx = \frac{1}{\pi} \cdot \left(\frac{x^3}{3}\right)_0^{2\pi} = \frac{8}{3}\pi^2$ ——

2(a). $f(x) = x^3 - 4x + 1$

$f(0) = 1 > 0$
$f(1) = -2 < 0$ } ∴ The root lies b/w ~~1 & 2~~ 0 and 1
$f(2) = 1 > 0$

let $x_0 = 0$; $x_1 = 1$

Ist approximation to the root is $x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} \cdot f(x_0)$ → 2m

$= 0 - \frac{(1-0)}{-2+1}(+1) = -1.666$. $0 - \frac{(1-0)}{-2-1} \cdot (1) = \frac{1}{3} = 0.33333$

$f(1.666) < 0$ ∴

The root lies b/w 1.666 and 2.

IInd approximation to the root is 1.8362 → 2m

Continue the same procedure → 1m

Ist appr to the root is

$x_2 = 0.3333$
$x_3 = 0.3478$
$x_4 = 0.3494$ } → 2m
$x_5 = 0.3495$

required root upto 3 decimals is 0.349 → 1m

2(b)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	
1	2	2	2	a−14	−4a+66	10a+b−222	−20a−6b+706
2	4	4	a−12	52−3a	6a+b−156	−10a−5b+484	15a+15b−1196
3	8	a−8	−2a+40	3a+b−104	−4a−4b+328	10b+5a−712	
4	a	32−a	a+b−64	−a−3b+224	6b+a−384		
5	32	b−32	160−2b	3b−160			
6	b	128−b	b				
7	128	128					
8	256						

→ 4m

$20a + 6b = 706$
$15a + 15b = 1196$ } solving these equation → 1m

$a = 16.2571$; $b = 63.476$.

(or) using shifting operator also give full marks.

(3)

Marks	No. of students	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
below 40	31	42	9	-25	37
" 50	73	51	-16	12	
" 60	124	35	-4		
" 70	159	31			
" 80	190				

→ 5M

$h = 10;\ m = \frac{a-c}{h} = 0.5$ → 1M

$y = y_0 + m\Delta y_0 + \frac{m(m-1)}{\lfloor 2}\Delta^2 y_0 + \ldots$ → 2M

$= 31 + (0.5)42 + \frac{(0.5)(0.5-1)}{\lfloor 2}(9) + \frac{(0.5)(0.5-1)(0.5-2)}{\lfloor 3}(-25)$

$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{\lfloor 4}(37)$ → 2M

$= 47.868.$

(4)

x	y	Δ	Δ^2	Δ^3			
1.0	7.989	0.414	-0.036	0.006	-0.002	0.001	0.002
1.1	8.403	0.378	-0.030	0.004	-0.001	0.003	
1.2	8.781	0.348	-0.026	0.003	0.002		
1.3	9.129	0.322	-0.023	0.005			
1.4	9.451	0.299	-0.018				
1.5	9.750	0.281					
1.6	10.031						

→ 4M

$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \ldots\right]$

$\left.\frac{dy}{dx}\right|_{1.1} = \frac{1}{0.1}\left[0.378 - \frac{1}{2}(-0.03) + \frac{1}{3}(0.004) - \frac{1}{4}(0.001) + \frac{1}{5}(0.003)\right]$ → 2M

$= 3.952$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 \cdots\cdots\right]$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=1.1} = \frac{1}{(0.1)^2}\left[-0.030-(0.004)+\frac{11}{12}(-0.001)-\frac{5}{6}(0.003)\right]$$

$$= -3.74. \longrightarrow 2m$$

$h=0.1$

$$\left.\frac{dy}{dx}\right|_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \cdots\right]$$

$$\left.\frac{dy}{dx}\right|_{x=1.6} = \frac{1}{0.1}\left[0.281+\frac{1}{2}(-0.0187)+\frac{1}{3}(0.005) + \frac{1}{4}(0.002)+\frac{1}{5}(0.003)+\frac{1}{6}(0.002)\right] \longrightarrow 1m$$

$$= 2.75$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n + \frac{137}{180}\nabla^6 y_n + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.018+0.005+\frac{11}{12}(0.002)+\frac{5}{6}(0.003) + \frac{137}{180}(0.002)\right] \longrightarrow 1m$$

$$= -0.715$$

5 (a) rate of velocity is acceleration : S. 1/3 rule —— 2m

$$v = \int_0^{80} a\,dt = \frac{10}{3}\left[(30+50.67)+4(31.63+35.47+40.33+46.69) + 2(33.34+37.75+43.25)\right] \longrightarrow 2m$$

$$= 30.86 \text{ m/sec} \text{ —— } 1m$$

5 (b)

y	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

$\longrightarrow$ 2m

By weddle's rule

$$\int_{0 = x_0}^{6 = x_n} \underbrace{\frac{1}{1+x^2}}_{f(x)}\,dx = \frac{3h}{10}\left[y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6\right] \longrightarrow 2m$$

$$= \frac{3\times 1}{10}\left[1+5(0.5)+0.2+6(0.1)+0.0588+5(0.0385)+0.027\right]$$

$$= 1.37355 \longrightarrow 1m$$

6. By using Picard's method (starting values)

$x_0 = 0;\quad y_0 = 0 \quad ; f_0 = 0$

$x_1 = 0.2 \quad y_1 = 0.02 \quad f_1 = 0.1996$

$x_2 = 0.4 \quad y_2 = 0.0795 \quad f_2 = 0.3937$

$x_3 = 0.6 \quad y_3 = 0.1762 \quad f_3 = 0.5689$

$x_4 = 0.8$

$x_5 = 1$

$\longrightarrow 4m$

Starting values y_1, y_2, y_3 by using any method give full marks.

$$y_4^{(p)} = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right] = 0.3049$$

$$f_4 = 0.7070$$

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4\right] = 0.3046.$$

$\longrightarrow 4m$

$$y_5^{(p)} = y_1 + \frac{4h}{3}\left[2f_2 - f_3 + 2f_4\right] = 0.4554$$

$$y_5^{(c)} = y_3 + \frac{h}{3}\left[f_3 + 4f_4 + f_5\right] = 0.4555$$

$\longrightarrow 2m$

7a. $\frac{dy}{dx} = x + y^2;\quad x_0 = 0;\ y_0 = 1;\ h = 0.1 \longrightarrow 1m$

$K_1 = hf(x_0, y_0) = (0.1)(0+1) = 0.1$

$K_2 = hf\left(x_0 + h/2,\ y_0 + \frac{K_1}{2}\right) = (0.1)(0.05 + 1.1025) = 0.11525$

$K_3 = hf\left(x_0 + h/2,\ y_0 + K_2/2\right) = (0.1)(0.05 + 1.11855) = 0.116855$

$\}\ 2m$

$$k_4 = hf(x_0+h, y_0+k_3)$$
$$=(0.1)\left[0.01+1.2474\right] = 0.1347$$
$$\Delta y = \frac{1}{6}\left[k_1+2k_2+2k_3+k_4\right]$$
$$= 0.1165$$

(2m)

$$\therefore y_1 = y_0+\Delta y = 1.1165$$

7(b) $x_0 = 1;\ y_0 = 2;\ h = 0.2$

E.M: $y_{m+1} = y_m + h f(x_m, y_m)$

$$y_1^{(0)} = y_0 + h f(x_0, y_0)$$
$$= 2 + 0.2\,(\log(x_0+y_0))$$
$$= 2+(0.2)(\log 3) = 2.0954 \longrightarrow \text{(2m)}$$

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0,y_0)+f(x_1, y_1^{(0)})\right]$$
$$= 2+\frac{0.2}{2}\left[\log(1+2)+\log(1.2+2.0954)\right]$$
$$= 2+0.0995 \longrightarrow \text{(2m)}$$
$$= 2.0995$$

$$y_1^{(2)} = y_0+\frac{h}{2}\left[f(x_0,y_0)+f(x_1,y_1^{(1)})\right]$$
$$= 2+\frac{0.2}{2}\left[\log(1+2)+\log(1.2+2.0995)\right] \longrightarrow \text{(1m)}$$
$$= \underline{2.0995}$$

8(a) $$L^{-1}\left\{\frac{4s}{(4s)^2-5^2}+\frac{15}{(4s)^2-5^2}\right\}$$

$$= L^{-1}\left\{\frac{4s}{16\left(s^2-\frac{25}{16}\right)}+\frac{15}{16\left(s^2-\frac{25}{16}\right)}\right\} \longrightarrow \text{(2m)}$$

$$= L^{-1}\left\{\frac{4}{16}\cdot\frac{s}{s^2-(5/4)^2}+\frac{15}{16}\left[\frac{1}{s^2-\left(\frac{5}{4}\right)^2}\right]\right\}$$

$$= \frac{1}{4}\left[L^{-1}\left(\frac{s}{s^2-(5/4)^2}\right)\right] + \frac{15}{16}\left\{L^{-1}\left(\frac{1}{s^2-(\frac{5}{4})^2}\right)\right\} \longrightarrow 2m$$

$$= \frac{1}{4}\cdot \cosh \tfrac{5}{4}t + \tfrac{3}{4}\cdot \sinh \tfrac{5}{4}t. \longrightarrow 1m$$

8(b) $L\{t\cdot e^t \sin t\}$: ?

$$L\{\sin t\} = \frac{1}{s^2+1} \longrightarrow 2m$$

$$L\{e^t \sin t\} = \frac{1}{(s-1)^2+1} \longrightarrow 2m$$

$$L\{t e^t \sin t\} = -\frac{d}{ds}\left[\frac{1}{s^2-2s+2}\right]$$

$$= \frac{2s-2}{(s^2-2s+2)^2} \longrightarrow 1m$$

9(a) $$L\{\cos at - \cos bt\} = \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2} \longrightarrow 2m$$

$$L\left\{\frac{\cos at - \cos bt}{t}\right\} = \int_s^\infty \left[\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right] ds$$

$$= \left[\frac{1}{2}\log(s^2+a^2) - \frac{1}{2}\log(s^2+b^2)\right]_s^\infty \longrightarrow 2m$$

$$= \frac{1}{2}\lim_{s\to\infty} \log\left[\frac{\cancel{s^2}(1+a^2/s^2)}{\cancel{s^2}(1+b^2/s^2)}\right] - \frac{1}{2}\log\left(\frac{s^2+a^2}{s^2+b^2}\right)$$

$$= 0 - \frac{1}{2}\log\left(\frac{s^2+a^2}{s^2+b^2}\right) \longrightarrow 1m$$

$$= \log\sqrt{\frac{s^2+b^2}{s^2+a^2}}.$$

9(b) let $F(s) = \log\left(\frac{s+1}{s-1}\right)$

$$L\{f(t)\} = F(s) = \log\left(\frac{s+1}{s-1}\right)$$

$$= \log(s+1) - \log(s-1) \longrightarrow 2m$$

$$L\{t\cdot f(t)\} = -\frac{d}{ds}\left[\log(s+1) - \log(s-1)\right]$$

$$= \frac{1}{s-1} - \frac{1}{s+1} \longrightarrow 2m$$

$$L\{t\cdot f(t)\} = L\{e^{t} - e^{-t}\}$$

$$f(t) = \frac{e^{t}-e^{-t}}{t} = \frac{2\sinh t}{t} \longrightarrow 1m$$

10. $f(-x) = x = f(x)$

$\therefore$ $f(x)$ is even function.

Required Fourier series is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \longrightarrow 2m$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left(\frac{\pi^2}{2}\right) = \pi \longrightarrow 2m$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x\cdot\left(\frac{\sin nx}{n}\right) - 1\cdot\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{1}{n^2}\left((-1)^n - 1\right)\right]$$

$= 0$ if n is even $\longrightarrow 2m$

$= -4/\pi n^2$ if n is odd.

$$\therefore f(x) = \pi/2 - \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right] \quad (1) \longrightarrow 2m$$

when $x = 0$; $|x| = |0| = 0$.

put $x = 0$ in (1).

$$f(0) = \pi/2 - \frac{4}{\pi}\left[\frac{\cos 0}{1^2} + \frac{\cos 0}{3^2} + \frac{\cos 0}{5^2} + \cdots\right]$$

$$-\pi/2 = -\frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

$$\Rightarrow \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \longrightarrow 2m$$

(11)

$$F\{f(x)\} = F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} \{a - |x|\}\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[a \cdot \int_{-a}^{a} e^{isx}\, dx + \int_{-a}^{0} x\, e^{isx}\, dx - \int_{0}^{a} x\, e^{isx}\, dx \right] \rightarrow (2m)$$

Now $I_1 = a \int_{-a}^{a} e^{isx}\, dx = \frac{a}{is}\left[e^{ias} - e^{-ias} \right]$ $\{s \neq 0\}$ $\rightarrow$ (2m)

$$I_2 = \int_{-a}^{0} x\, e^{isx}\, dx = \frac{1}{is}\left[-a \cdot e^{-ias} \right] + \frac{1}{s^2}\left(1 - e^{-ias}\right) \rightarrow (2m)$$

$$I_3 = \int_{0}^{a} x\, e^{isx}\, dx = \frac{a}{is} e^{ias} + \frac{1}{s^2}\left(e^{ias} - 1\right)$$

$\therefore F(s) = \frac{1}{\sqrt{2\pi}} \left[I_1 + I_2 + I_3 \right]$ by simplifying

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \left[\frac{1 - \cos as}{s^2} \right]. \rightarrow (2m)$$

The inverse Fourier transform of $f(x)$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) \cdot e^{isx}\, dx.$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} \sqrt{\frac{2}{\pi}} \left(\frac{1 - \cos as}{s^2} \right) \left(\cos sx - i \sin sx \right) ds$$

equating the real and imaginary parts.

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \left(\frac{1 - \cos as}{s^2} \right) \cos sx\, dx = f(x)$$

The integrand is even fn $\therefore \frac{2}{\pi} \int_{0}^{\infty} \left(\frac{1 - \cos as}{s^2} \right) \cos sx\, dx = f(x)$

$$\therefore \int_{0}^{\infty} \left(\frac{1 - \cos as}{s^2} \right) \cos sx\, dx = \pi/2\, f(x)$$

$$= \pi/2 \{ a - |x| \; ; \; x < a \}$$

$$= 0 \; ; \; |x| > a$$

Put $a = 2$; and $x = 0$. $\rightarrow$ (2m)

$$\therefore \int_{0}^{\infty} \left(\frac{\sin s}{s} \right)^2 ds = \pi/2$$