	b)	Using Cauchy's integral formula, evaluate $\oint_C \frac{e^z}{z(1-z)^3}\,dz$, where C is $\lvert z\rvert = \frac{1}{2}$	L4	CO5	5 M
		OR			
9	a)	Expand the Laurent series of $f(z) = \frac{1}{z^2-3z+2}$ in the region $0 < \lvert z-1\rvert < 1$	L3	CO3	5 M
	b)	Find the Taylor's series expansion of the function $\frac{2z^3+1}{z^2+z}$ about the point z=1.	L3	CO3	5 M
		UNIT-V			
10	a)	Determine and classify the singularities of $\frac{z}{e^z-1}$	L3	CO3	5 M
	b)	Evaluate $\int_C \frac{5z-2}{z(z-1)}\,dz$ where C is $\lvert z\rvert = 2$	L3	CO3	5 M
		OR			
11		Evaluate $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$ using contour integration in the complex plane.	L4	CO5	10 M

Code: 23BS1302

II B.Tech - I Semester – Regular / Supplementary Examinations
NOVEMBER 2025

NUMERICAL METHODS AND COMPLEX VARIABLES
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Prove that $E\nabla = \Delta = \nabla E$	L2	CO1
1.b)	Evaluate $\Delta(e^{2x+3})$	L2	CO1
1.c)	Write the formula of first derivative of the function $y = f(x)$ at $x = x_n$ using Newton's backward Interpolation formula.	L3	CO2
1.d)	State Eulers's method formula.	L2	CO1
1.e)	Write the Cauchy – Riemann (C-R) equations in cartesian form.	L2	CO1
1.f)	Prove that the function $u(x,y) = 3xy^2 - x^3$ is harmonic.	L3	CO3
1.g)	Expand ze^z as Maclaurin's series.	L3	CO3
1.h)	Evaluate $\int_0^{1+i} (x^2 - iy)\,dz$ along the path $y = x$	L2	CO1

1.i)	Write the zero's and the poles of $f(z)=\frac{z^4+z^3}{z^3(1-z)}$	L3	CO3
1.j)	Determine the residue of the function $\frac{9z+1}{z(z^2+1)}$ at z=0	L3	CO3

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2	a)	Find the missing values in the following table x: 0, 5, 10, 15, 20, 25 y: 6, 10, --, 17, --, 31	L4	CO4	5 M
	b)	Apply bisection method to find a real root of the equation $x^3-4x-9=0$.	L3	CO2	5 M
		OR			
3	a)	Estimate the population in the year 1895 using Newton's forward interpolation formula from the following statistics: Year x: 1891, 1901, 1911, 1921, 1931 Population y: 46, 66, 81, 93, 101	L4	CO4	5 M
	b)	Estimate the value of y at x=10 using Lagrange's interpolation formula from the following table x: 5, 6, 9, 11 y: 12, 13, 14, 16	L4	CO4	5 M

Tables within Q2 a), Q3 a), Q3 b):

x	0	5	10	15	20	25
y	6	10	--	17	--	31

Year x:	1891	1901	1911	1921	1931
Population y:	46	66	81	93	101

x	5	6	9	11
y	12	13	14	16

		UNIT-II			
4	a)	Using forward difference formulae, obtain $\frac{dy}{dx}$ at $x=2$ for the data given below: x: 2, 4, 6, 8, 10 y: 0, 0, 1, 0, 0	L3	CO2	5 M
	b)	Evaluate $\int_0^1 x^3\,dx$ with five subintervals taking h=0.2 by trapezoidal rule.	L4	CO4	5 M
		OR			
5		Estimate $y(0.2)$ in steps of 0.1 using Runge-Kutta 4th order (R-K) method, given that $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2},\ y(0)=1$.	L4	CO4	10 M
		UNIT-III			
6	a)	Prove that $\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\lvert\text{Real } f(z)\rvert^2=2\lvert f'(z)\rvert^2$ where $w=f(z)=u+iv$ is analytic.	L4	CO5	5 M
	b)	Show that $v(x,y)=-\sin x\sinh y$ is harmonic. Find the conjugate harmonic of $v(x,y)$ where $w=f(z)=u+iv$ is analytic.	L3	CO3	5 M
		OR			
7		Construct an analytic function $f(z)=u+iv$ whose real part is $u=e^x(x\sin y+y\cos y)$	L3	CO3	10 M
		UNIT-IV			
8	a)	Evaluate $\oint_C z^2\,dz$ around the square with the vertices at (0,0), (1,0),(1,1), (0,1)	L4	CO5	5 M

Table within Q4 a):

x:	2	4	6	8	10
y:	0	0	1	0	0

Subject Code: 23BS1302

PVP23

II B.Tech. I Semester Regular Examinations - NOVEMBER-2025

NUMERICAL METHODS and COMPLEX VARIABLES
(ELECTRICAL and ELECTRONICS ENGINEERING)

Key & Scheme of Evaluation

1.a)	We know that $\Delta = E-1$ and $\nabla = 1-E^{-1}$ Then $E\nabla = E(1-E^{-1}) = E-1 = \Delta$ $\nabla E = (1-E^{-1})E = E-1 = \Delta$	1 1
1.b)	$\Delta(e^{2x+3}) = e^{2(x+h)+3} - e^{2x+3}$ $= e^{2(x+h)+3} - e^{2x+3} = e^{2x+3}(e^{2h}-1)$	1 1
	Note:-Marks can be awarded for alternate procedure(s) and/or solution(s) taking any value for h	
1.c)	$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n - \cdots\cdots\right]$	2
1.d)	$y_{n+1} = y_n + h.f(x_n, y_n)$ for $n = 0,1,2,\cdots$ **or** $y_1 = y_0 + h.f(x_0, y_0)$ $y_2 = y_1 + h.f(x_1, y_1)$	2
1.e)	$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ $(u_x = v_y)$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ $(v_x = -u_y)$	1 1
1.f)	$u_x = 3y^2 - 3x^2$, $u_{xx} = -6x$ $u_y = 6xy$, $u_{yy} = 6x$ Then, $u_{xx} + u_{yy} = 0 \Rightarrow u$ is harmonic.	1 1
1.g)	$ze^z = z\left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!}\cdots\cdots\right)$ $= \sum_{n=0}^{\infty} \frac{z^{n+1}}{n!}$	2
	Note:-Marks can be awarded for alternate procedure(s) and/or solution(s)	
1.h)	Along $y = x$, $z = x + iy = (1+i)x \Rightarrow dz = (1+i)dx$ $\int_0^{1+i} (x^2 - iy)dz = \int_0^1 (x^2 - ix)(1+i)dx = (1+i)\left(\frac{1}{3} - \frac{i}{2}\right) = \frac{1}{6}(5-i)$	1 1
1.i)	$f(z) = \frac{z^4+z^3}{z^3(1-z)} = \frac{z}{1-z}$ Zeros of $f(z)$ 0 Poles of $f(z)$ 1 **or** Zeros of $f(z)$ are $0, -1$ Poles of $f(z)$ are $0, 1$	1 1
1.j)	Res$f(0) = \lim_{z\to 0} zf(z)$ $= \lim_{z\to 0} \frac{9z+1}{z^2+1} = 1$	1 1
2.a)	$x_0 = 0$ $x_1 = 5$ $x_2 = 10$ $x_3 = 15$ $x_4 = 20$ $x_5 = 25$ $y_0 = 6$ $y_1 = 10$ y_2 $y_3 = 17$ y_4 $y_5 = 31$ $\Delta^4 y = 0, \forall y$ In particular, $\Delta^4 y_0 = 0$ and $\Delta^4 y_1 = 0$ $y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$ and $y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$ $\Rightarrow y_4 + 6y_2 - 102 = 0$ and $-4y_4 - 4y_2 + 143 = 0$ $\Rightarrow y_4 = 22.5$ and $y_2 = 13.25$	2 2 1
	Note:-Marks can be awarded for alternate procedure(s) and/or solution(s)	

2.b)

Let $f(x) = x^3 - 4x - 9 = 0$

$f(2.7) = -0.117 < 0$

$f(2.8) = 1.752 > 0$ **2**

Root lies between 2.7 and 2.8

a	b	$f(a)$	$f(b)$	$c = \frac{a+b}{2}$	$f(c)$
2.7	2.8	-0.117	1.752	2.75	0.7969
2.7	2.75	-0.117	0.7969	2.725	0.3348
2.7	2.725	-0.117	0.3348	2.7125	0.1076
2.7	2.7125	-0.117	0.1076	2.7063	-0.005
2.7063	2.7125	-0.005	0.1076	2.7094	

2

$\therefore$ 2.7094 is a root for the given equation corrected to two decimal places. **1**

Note:-Marks can be awarded for alternate procedure(s) or values (s) and/or root(s) or solution(s)

3.a)

x	y	Δ	Δ^2	Δ^3	Δ^4
x_0 **1891**	y_0 **46**				
		20			
x_1 1901	y_1 66		**-5**		
		15		**2**	
x_2 1911	y_2 81		-3		**-3**
		12		-1	
x_3 1921	y_3 93		-4		
		8			
x_4 1931	y_4 101				

2

By Newton's forward interpolation formula

$$y = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\,\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\,\Delta^3 y_0 + \cdots\cdots\cdots$$ **1**

where $p = \frac{x-x_0}{h} = \frac{1895-1891}{10} = 0.4$

$$= 46 + 0.4(20) + \frac{0.4(-0.6)}{2}(-5) + \frac{0.4(-0.6)(-1.6)}{6}(2) + \frac{0.4(-0.6)(-1.6)(-2.6)}{24}(-3)$$

$= 54.85$ **2**

3.b) Let $x_0 = 5 \quad x_1 = 6 \quad x_2 = 9 \quad x_3 = 11$

$y_0 = 12 \quad y_1 = 13 \quad y_2 = 14 \quad y_3 = 16$ **1**

Lagrange's Interpolation formula,

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\,y_0 + \cdots\cdots\cdots + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\,y_3$$ **2**

$$= \frac{(4)(1)(-1)}{(-1)(-4)(-6)}(12) + \frac{(5)(1)(-1)}{(1)(-3)(-5)}(13) + \frac{(5)(4)(-1)}{(4)(3)(-2)}(14) + \frac{(5)(4)(1)}{(6)(5)(2)}(16)$$ **1**

$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = 14.67$ **1**

4.a)

x	y	Δ	Δ^2	Δ^3	Δ^4
x_0 **2**	y_0 **0**				
		0			
x_1 4	y_1 0		**1**		
		1		**-3**	
x_2 6	y_2 1		-2		**6**
		-1		3	
x_3 8	y_3 **0**		1		

0

x_4 10 $\quad$ y_4 0 **2**

$$\left(\frac{dy}{dx}\right)_{x=2} = \frac{1}{h}\left(\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\cdots\right)$$ **2**

$$= \frac{1}{2}\left[0 - \frac{1}{2}(1) + \frac{1}{3}(-3) - \frac{1}{4}(6)\right] = -\frac{3}{2} = -1.5$$ **1**

4.b) Here step size $h = 0.2$

x	0	0.2	0.4	0.6	0.8	1
$y = x^3$	0	0.008	0.064	0.216	0.512	1
	y_0	y_1	y_2	y_3	y_4	y_5

2

By Trapezoidal rule, $\int_0^1 y\, dx = \frac{h}{2}\,[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$ **2**

$$= \frac{(0.2)}{2}\,[(0 + 1) + 2(0.008 + 0.064 + 0.216 + 0.512)]$$

$= 0.26$ **1**

5) Given $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \ni y(0) = 1$

Here $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$, $x_0 = 0$, $y_0 = 1$ $\quad$ $h = 0.1$ **2**

$x_1 = x_0 + h = 0.1$ $\quad$ $x_2 = x_1 + h = 0.2$

To find y_1:

$k_1 = h.f(x_0\,,\ y_0) = (0.1).f(0\,,1) = 0.1$

$k_2 = h.f\left(x_0 + \frac{h}{2}\,, y_0 + \frac{k_1}{2}\right) = (0.1).f(0.05\,,1.05) = 0.0995$

$k_3 = h.f\left(x_0 + \frac{h}{2}\,, y_0 + \frac{k_2}{2}\right) = (0.1).f(0.05\,,1.0498) = 0.0995$ **3**

$k_4 = h.f(x_0 + h\,, y_0 + k_3) = (0.1).f(0.1, 1.0995) = 0.0984$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.0994$$

$\therefore y_1 = y(0.1) = y_0 + k = 1.0994$ **1**

To find y_2:

$k_1 = h.f(x_1\,, y_1) = (0.1).f(0.1\,,1.0994) = 0.0984$

$k_2 = h.f\left(x_1 + \frac{h}{2}\,, y_1 + \frac{k_1}{2}\right) = (0.1).f(0.15\,,1.1486) = 0.0966$

$k_3 = h.f\left(x_1 + \frac{h}{2}\,, y_1 + \frac{k_2}{2}\right) = (0.1).f(0.15\,,1.1477) = 0.0966$

$k_4 = h.f(x_1 + h\,, y_1 + k_3) = (0.1).f(0.2\,,1.1961) = 0.0946$ **3**

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.0966$$

$\therefore y_2 = y_1 + k = 1.196$ **1**

6.a) Given, $f(z) = u + iv$ is an analytic function.

Then $f(z)$ satisfies C-R equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

Here, $\mathrm{R}[f(z)] = u \Longrightarrow |\,\mathrm{R}[f(z)]\,|^2 = u^2$ **2**

Now $\frac{\partial}{\partial x}\left(|\,\mathrm{R}[f(z)]\,|^2\right) = \frac{\partial}{\partial x}(u^2) = 2u\frac{\partial u}{\partial x}$

$$\Longrightarrow \frac{\partial^2}{\partial x^2}\left(|\,\mathrm{R}[f(z)]\,|^2\right) = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(|\,\mathrm{R}[f(z)]\,|^2\right)\right] = \frac{\partial}{\partial x}\left[2u\frac{\partial u}{\partial x}\right]$$

$$= 2\left[\left(\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial x} + u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)\right)\right] = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + u\frac{\partial^2 u}{\partial x^2}\right] \longrightarrow \boxed{1}$$ **1**

Similarly, $\frac{\partial^2}{\partial y^2}\left(|\,\mathrm{R}[f(z)]\,|^2\right) = 2\left[\left(\frac{\partial u}{\partial y}\right)^2 + u\frac{\partial^2 u}{\partial y^2}\right] \longrightarrow \boxed{2}$ **1**

	$\boxed{1}+\boxed{2} \Rightarrow \frac{\partial^2}{\partial x^2}\left(\mid \mathrm{R}[f(z)] \mid^2\right)+\frac{\partial^2}{\partial y^2}\left(\mid \mathrm{R}[f(z)] \mid^2\right)$ $= 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + u\frac{\partial^2 u}{\partial x^2}\right] + 2\left[\left(\frac{\partial u}{\partial y}\right)^2 + u\frac{\partial^2 u}{\partial y^2}\right]$ $= 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\right]$ $= 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + u(0)\right]$ $f'(z) = u_x + iv_x \Rightarrow \lvert f'(z)\rvert = \sqrt{u_x^2+v_x^2}$ $= 2\lvert f'(z)\rvert^2$	1
6.b)	$v = -\sin x \sinh y$ Then $\frac{\partial v}{\partial x} = -\cos x \sinh y \;\Rightarrow\; \frac{\partial^2 v}{\partial x^2} = \sin x \sinh y$ $\frac{\partial v}{\partial y} = -\sin x \cosh y \;\Rightarrow\; \frac{\partial^2 v}{\partial y^2} = -\sin x \sinh y$ $\therefore\; \frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} = 0$	2
	Since, $f(z) = u+iv$ is an analytic function, $f(z)$ satisfies C-R equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ $\therefore f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$ $= (-\sin x \cosh y) + i(-\cos x \sinh y)$	2
	By Milne-Thomson's method, replace x by z and y by 0 to obtain $f'(z) = (-\sin z) - i(0)$ $\Rightarrow f(z) = \cos z + A \;\Rightarrow\; u = \cos x \cosh y + a$	1
7)	$f(z) = u+iv$ be an analytic function where $v(x,y) = e^x(x\sin y + y\cos y)$ Then $\frac{\partial v}{\partial x} = e^x(x\sin y + y\cos y + \sin y)$ and	2
	$\frac{\partial v}{\partial y} = e^x(x\cos y + \cos y - y\sin y)$	2
	Since, $f(z)$ is an analytic function, $f(z)$ satisfies C-R equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ $\therefore f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$	2
	$= e^x(x\cos y + \cos y - y\sin y) + ie^x(x\sin y + y\cos y + \sin y)$	2
	By Milne-Thomson's method, replace x by z and y by 0 to obtain $f'(z) = e^z(z+1) - i(0)$ $\Rightarrow f(z) = ze^z + A$	2
	Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).	
8.a)	Clearly, $f(z) = z^2$ is an analytic function everywhere around a closed curve(square) .	3
	$\therefore$ By Cauchy's theorem, $\int_C f(z)\,dz = 0$	2
	Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).	
8.b)	Clearly $z = 0, 1$ are singular points of $\frac{e^z}{z(1-z)^3}$ of which $z = 0$ only lies inside $C: \lvert z\rvert = \frac{1}{2}$.	2
	Here $f(z) = \frac{e^z}{(1-z)^3}$, $a = 0$	1
	By Cauchy's integral formula, $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z-a}dz$	1
	$\Rightarrow f(0) = \frac{1}{2\pi i}\int_C \frac{\left[\frac{e^z}{(1-z)^3}\right]}{z}dz$ $\Rightarrow \int_C \frac{e^z}{z(1-z)^3}dz = 2\pi i \cdot f(0) = 2\pi i(1) = 2\pi i$	1

	Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).	
9.a)	$f(z)=\frac{1}{z^2-3z+2}=\frac{1}{(z-1)(z-2)}=\frac{1}{z-2}-\frac{1}{z-1}$	2
	$\underline{\lvert z-1\rvert<1:}\quad f(z)=\frac{1}{z-2}-\frac{1}{z-1}$ $=\frac{1}{-[1-(z-1)]}-\frac{1}{z-1}$	1
	$=-[1-(z-1)]^{-1}-\frac{1}{z-1}$	1
	$=-[1+(z-1)+(z-1)^2+\cdots\cdots]-\frac{1}{z-1}$	1
	Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).	
9.b)	$\frac{2z^3+1}{z^2+z}=2z-2+\frac{1}{z}+\frac{1}{z+1}$	2
	$=2(z-1)+\frac{1}{[1+(z-1)]}+\frac{1}{[2+(z-1)]}$	1
	$=2(z-1)+[1+(z-1)]^{-1}+\frac{1}{2}\left[1+\frac{(z-1)}{2}\right]^{-1}$	1
	$=2(z-1)+[1-(z-1)+(z-1)^2-\cdots\cdots]+\frac{1}{2}\left[1-\left(\frac{z-1}{2}\right)+\left(\frac{z-1}{2}\right)^2-\cdots\cdots\right]$	1
	Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).	
10.a)	Let $f(z)=\frac{z}{e^z-1}$ $z=0,2in\pi$ are singular points of $f(z)$ for all n non-zero integer.	2
	$\underline{z=0:}\quad lim_{z\to 0}\, f(z)=\lim_{z\to 0}\frac{z}{e^z-1}=1\neq 0$ $\therefore z=\ 1$ is a removable singularity.	2
	$\underline{z=2in\pi:}\quad \lim_{z\to 2in\pi}(z-2in\pi)f(z)\neq 0$ $\therefore z=\ 2in\pi$ is a simple pole.	1
	Note:- Marks can be awarded for alternate procedure(s) or method(s) or solution(s).	
10.b)	Let $f(z)=\frac{5z-2}{z(z-1)}$ Clearly, $z=0,1$ are singular points and both lies inside the circle $C:\lvert z\rvert=2$	1
	$\lim_{z\to 0} zf(z)=\lim_{z\to 0}\frac{5z-2}{z-1}=2\neq 0$ $\therefore z=\ 0$ is a simple pole. Then, Res$f(0)=lim_{z\to 0}\,(z)f(z)=2$ $\lim_{z\to 1}(z-1)f(z)=\lim_{z\to 1}\frac{5z-2}{z}=3\neq 0$ $\therefore z=1$ is a simple pole. Then, Res$f(1)=lim_{z\to 1}\,(z-1)f(z)=3$	2
	$\therefore$ By Cauchy-Residue theorem, $\int_C f(z)dz=2\pi i\,[\text{Sum of residues at singular points within C}]$	1
	$=2\pi i\,[\text{Res}f(0)+\text{Res}f(1)]$ $=2\pi i\,[2+3]=10\pi i$	1
11)	Put $z=e^{i\theta}$. Then $\cos\theta=\frac{z^2+1}{2z}$, $d\theta=\frac{dz}{iz}$ $\therefore\ \int_0^{2\pi}\frac{d\theta}{3+2\cos\theta}=\int_C\frac{(dz/iz)}{\left[3+2\left(\frac{z^2+1}{2z}\right)\right]}$	2

	$= \frac{2}{i}\int_C \frac{dz}{2z^2+6z+2}$ $= \frac{1}{i}\int_C \frac{dz}{z^2+3z+1} = \frac{1}{i}\int_C f(z)dz$ where $f(z) = \frac{1}{z^2+3z+1}$	2
	The singular points of $f(z)$ are $z = \alpha,\ \beta$ where $\alpha = \frac{-3+\sqrt{5}}{2},\ \beta = \frac{-3-\sqrt{5}}{2}$ $\lvert\alpha\rvert < 1$ and $\lvert\beta\rvert > 1$	2
	Also $z = \alpha$ is a simple pole, Res$f(\alpha) = lim_{z\to\alpha}(z-\alpha)f(z)$ $= \lim_{z\to\alpha}(z-\alpha)\cdot\frac{1}{(z-\alpha)(z-\beta)}$ $= \frac{1}{\alpha-\beta} = \frac{1}{\sqrt{5}}$	1
	∴ By Cauchy-Residue theorem, $\int_C f(z)dz = 2\pi i\ [\text{Sum of residues at singular points within C}]$ $= 2\pi i\ [\text{Res}f(\alpha)] = 2\pi i\left[\frac{1}{\sqrt{5}}\right] = \frac{2\pi i}{\sqrt{5}}$	2
	$\therefore \int_0^{2\pi}\frac{d\theta}{3+2\cos\theta} = \frac{1}{i}\int_C f(z)dz = \frac{1}{i}\left(\frac{2\pi i}{\sqrt{5}}\right) = \frac{2\pi}{\sqrt{5}}$	1

Note:- Marks can be awarded for alternate procedure(s) or method(s) and/or solution(s) and/or root(s).