OR					
11	a)	State and explain Maxwell's fourth equation.	L2	CO5	5 M
	b)	Explain the Poynting theorem in detail.	L2	CO5	5 M

Code: 23ES1301

II B.Tech - I Semester – Regular / Supplementary Examinations
NOVEMBER 2025

ELECTROMAGNETIC FIELD THEORY
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Express the importance of Coulomb's law?	L2	CO1
1.b)	Define the concept of electric field intensity.	L1	CO1
1.c)	Define electric potential and potential difference.	L1	CO1
1.d)	Classify the applications of Laplace's equation?	L2	CO1
1.e)	Explain Magnetic dipole.	L2	CO3
1.f)	Express point form of Ampere's circuital law.	L2	CO3
1.g)	Express the relation between magnetic flux and magnetic flux density.	L2	CO3
1.h)	Define self and mutual inductances.	L1	CO4
1.i)	List the significance of displacement current.	L1	CO4
1.j)	Define the dynamically induced EMF.	L1	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2		Derive an expression for the electrical field intensity due to an infinite line charge and a circular disk charge distribution.	L3	CO1	10 M
OR					
3	a)	State and explain Maxwell's first equation.	L2	CO1	5 M
	b)	A point charge Q_1=2μC located at origin and another point charge Q_2=-5μC is at (3,4,0) m. Calculate the electric force on charge Q_1.	L3	CO1	5 M
UNIT-II					
4		Three-point charges, Q_1 = 1 mC, Q_2 = -2 mC, and Q_3 = 3 mC, are located at (0, 0, 4), (−2, 5, 1), and (3, −4, 6), respectively. (i) Find the electric field E and potential V_P at P (−1, 1, 2). (ii) Calculate the potential difference V_{PQ} if Q is (1, 2, 3).	L3	CO1	10 M
OR					
5	a)	Obtain an expression for the electric potential due to an electric dipole.	L2	CO2	5 M
	b)	Explain the concept of an electric dipole and dipole moment in detail.	L2	CO2	5 M

UNIT-III					
6	a)	Explain Biot-Savart's law in detail.	L2	CO3	5 M
	b)	Derive an expression for the magnetic field intensity due to an infinite sheet.	L3	CO3	5 M
OR					
7	a)	Produce the expressions for the capacitance of a parallel plate capacitor and the energy stored in it.	L3	CO2	5 M
	b)	Two current elements $I_1 dl_1 = 4\times10^{-5} a_x$ A.m at (0, 0, 0) and $I_2 dl_2 = 6\times10^{-5} a_y$ A.m at (0, 0, 1) are in free space. Calculate H at (3, 1,−2).	L3	CO3	5 M
UNIT-IV					
8	a)	Discuss the self-inductance of a solenoid with a neat schematic diagram.	L2	CO4	5 M
	b)	Summarize the force between two long and parallel current-carrying conductors in a magnetic field.	L2	CO4	5 M
OR					
9	a)	Solve the self-inductance of a coaxial cable with a neat schematic diagram.	L3	CO4	5 M
	b)	Derive an expression for the force on a current element in a magnetic field.	L3	CO3	5 M
UNIT-V					
10	a)	Solve an expression for the statically induced EMF in a time-varying field.	L3	CO5	5 M
	b)	Compare static fields and time-varying fields.	L4	CO5	5 M

PVP23

II B.Tech I Semester Regular/ Supplementary Examinations
November 2025

Electromagnetic Field Theory 23ES1301
(Electrical & Electronics Engineering)

Scheme

2		Equations	5M
		Derivations	5M
3	a)	Statement	2M
		Explanation	3M
	b)	Force equation	2M
		Calculation	3M
4	a)	E equation	2M
		V equation	2M
		Calculation V_P	3M
		Calculation V_{PQ}	3M
5	a)	Electric Dipole	2M
		Derivation	3M
	b)	Explanation	3M
		Dipole moment	2M
6	a)	Explanation	5M
	b)	Diagram	2M
		Derivation	3M
7	a)	Derivation	3M
		Equations	2M
	b)	Equations	3M
		Calculation	2M
8	a)	Diagram	3M
		Derivation	2M
	b)	Diagram	2M
		Derivation	3M
9	a)	Diagram	2M
		Explanation	3M
	b)	Derivation	5M
10	a)	Derivation	5M
	b)	Comparison	5M
11	a)	Statement	2M
		Explanation	3M
	b)	Explanation	5M

Part A

1

a)

Coulomb's law is important because it quantifies the electrostatic force between charged particles, serving as a foundation for the entire theory of electromagnetism and explaining phenomena from atomic structures to everyday static electricity. Its importance lies in its ability to predict and calculate forces in a vast range of applications, including the attraction between a nucleus and electrons, the behavior of molecules, the design of electronic components like capacitors, and the operation of advanced machinery such as particle accelerators and mass spectrometers.

b)

Force per unit charge $E=F/q$

c)

Electric potential is the amount of work done to move a unit positive charge from infinity to a specific point in an electric field. Potential difference, also known as voltage, is the difference in electric potential between two points and represents the work required to move a unit charge from one point to the other.

d)

Applications of Laplace's Equation

Physics, Engineering, Computer Science, Wave Phenomena etc

e)

A magnetic dipole is a pair of equal and opposite magnetic poles, a north and a south pole, separated by a small distance.

f)

The point form of Ampère's circuital law is $\nabla\times\vec{H}=\vec{J}$

g)

$B = \mu_o H$

h)

Self-inductance is a property of a single coil that induces an electromotive force (EMF) within itself when the current through it changes. Mutual inductance is the property of two nearby coils where a changing current in one coil induces an EMF in the other.

i)

Consistency of Maxwell's equations, Enables electromagnetic waves, Explains behavior in capacitors, Foundation for modern technology

j)

Dynamically induced electromotive force (EMF) is a voltage generated in a conductor when it moves through a magnetic field, causing it to cut magnetic flux lines.

Part B

2

Line charge

Consider an infinitely long straight line carrying uniform line charge having density ρ_L C/m. Let this line lies along z-axis from -∞ to ∞ and hence called **infinite line charge**. Let point P is on y-axis at which electric field intensity is to be determined. The distance of point P from the origin is 'r' as shown in the Fig. 2.6.1.

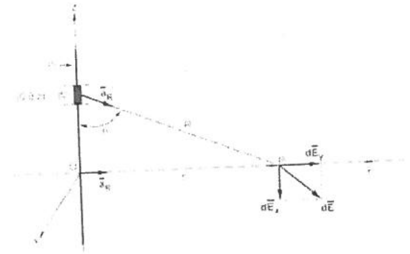


Fig. 2.6.1 Field due to infinite line charge

- Consider a small differential length dZ carrying a charge dQ, along the line as shown in the Fig. 2.6.1. It is along z-axis hence dZ = dz. $dQ = \rho_L\, dl = \rho_L\, dz$ ... (2.6.1)
- The co-ordinates of dQ are (0, 0, z) while the co-ordinates of point P are (0, r, 0). Hence the distance vector $\overline{R}$ can be written as,

$$\overline{R} = \overline{r}_P - \overline{r}_{dl} = \left[r\,\overline{a}_y - z\,\overline{a}_z \right]$$

$$\therefore \quad |\overline{R}| = \sqrt{r^2 + z^2}$$

$$\therefore \quad \overline{a}_R = \frac{\overline{R}}{|\overline{R}|} = \frac{r\,\overline{a}_y - z\,\overline{a}_z}{\sqrt{r^2 + z^2}} \qquad \text{... (2.6.2)}$$

$$\therefore \quad d\overline{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\,\overline{a}_R$$

$$= \frac{\rho_L\, dz}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2}\right)^2} \left[\frac{r\,\overline{a}_y - z\,\overline{a}_z}{\sqrt{r^2 + z^2}} \right] \qquad \text{... (2.6.3)}$$

Note: For every charge on positive z-axis there is equal charge present on negative z-axis. Hence the z component of electric field intensities produced by such charges at point P will cancel each other. Hence effectively there will not be any z component of $\overline{E}$ at P. This is shown in the Fig. 2.6.2.

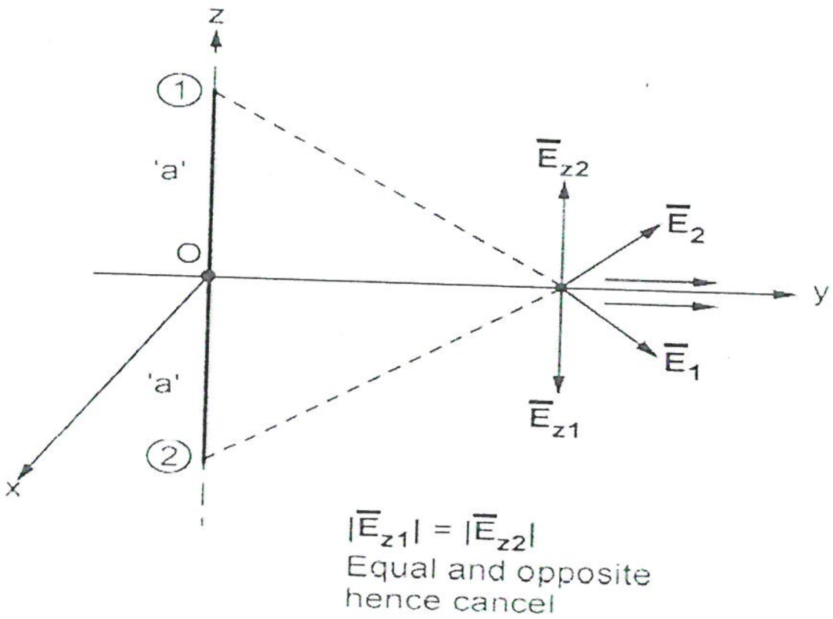


Fig. 2.6.2

- Hence the equation of $d\bar{E}$ can be written by eliminating $\bar{a}_z$ component,

$$\therefore \quad d\bar{E} = \frac{\rho_L \, dz}{4\pi\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^2} \frac{r\,\bar{a}_y}{\sqrt{r^2+z^2}} \qquad \text{... (2.6.4)}$$

- Now by integrating $d\bar{E}$ over the z axis from $-\infty$ to ∞ we can obtain total $\bar{E}$ at point P.

$$\therefore \quad \bar{E} = \int_{-\infty}^{\infty} \frac{\rho_L}{4\pi\varepsilon_0 \left(r^2+z^2\right)^{3/2}} \, r\,dz\,\bar{a}_y$$

Note : For such an integration, use the substitution
$z = r\tan\theta$ i.e. $r = z/\tan\theta$
$dz = r\ \sec^2\theta\ d\theta$
- Here r is not the variable of integration.

For $z = -\infty$, $\theta = \tan^{-1}(-\infty) = -\pi/2 = -90°$
For $z = +\infty$, $\theta = \tan^{-1}(\infty) = \pi/2 = +90°$

$$\therefore \bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L}{4\pi\varepsilon_0\left[r^2 + r^2\tan^2\theta\right]^{3/2}} r \times r\sec^2\theta \, d\theta \, \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2\sec^2\theta \, d\theta}{r^3\left[1+\tan^2\theta\right]^{3/2}} \bar{a}_y$$

But $1 + \tan^2\theta = \sec^2\theta$

$$\therefore \bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{r\sec^3\theta} \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \bar{a}_y \quad \text{changing the limits}$$

$$\ldots \sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} [\sin\theta]_{-\pi/2}^{\pi/2} \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \left[\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right] \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} [1-(-1)] \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \times 2 \, \bar{a}_y$$

$$\therefore \quad \bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_y \quad \text{V/m} \qquad \ldots (2.6.5)$$

Key Point : If without considering symmetry of charges and without cancelling z component from d $\bar{E}$, if integration is carried out, it gives the same answer. The integration results the z component of $\bar{E}$ to be mathematically zero.

- The result of equation (2.6.5) which is specifically in cartesian system can be generalized.

The $\bar{a}_y$ is unit vector along the distance r which is perpendicular distance of point P from the line charge. Thus in general $\bar{a}_y = \bar{a}_r$.

- Hence the result of $\bar{E}$ can be expressed as,

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_r \quad \text{V/m} \qquad \ldots (2.6.6)$$

Circular Disc

The electric field intensity (E) on the axis of a uniformly charged circular disk is given by the $E = \sigma/2\epsilon_0 (1 - x/\sqrt{R^2+x^2})$, where σ is the surface charge density, ϵ_0 is the permittivity of R is the disk's radius, and x is the distance from the disk's center along the axis.

Formula: $E=\sigma^2/\epsilon_0\,(1-x/\sqrt{R^2+x^2})$

σ: Surface charge density (charge per unit area)

$\epsilon 0$: Permittivity of free space ($8.854\times10^{-12} C^2/N\cdot m^2$)

R: Radius of the disk

x: Distance from the center of the disk along its axis

Key characteristics of the field

Maximum field: The electric field is maximum at the center of the disk (x=0).

Field near the disk: As the distance x becomes very small compared to the radius R, the formula simplifies to $E \approx \sigma/2\epsilon_0$, similar to the field of an infinite sheet of charge.

Field far from the disk: As the distance x becomes much larger than the radius R, the formula approaches the electric field of a point charge located at the disk's center.

3

a)

Maxwell's first equation, derived from Gauss's Law for electricity, states that the total electric flux through a closed surface is equal to the enclosed charge. Its differential form is $\nabla\cdot\vec{D}=\rho$, where the divergence of the electric displacement field ($\vec{D}$) equals the volume charge density (ρ). This means that electric field lines originate from positive charges and terminate on negative charges, and their net flow through any closed surface is determined by the total charge inside that surface.
that electric field lines originate from positive charges and terminate on negative charges, and their net flow through any closed surface is determined by the total charge inside that surface.

Explanation of the equation

- Integral Form: $\oint_S \vec{D}\cdot dS = \int_V \rho dV$

  $\oint S\vec{D}\cdot d\vec{A}$: This represents the total electric flux (or displacement flux) passing through a closed surface S.

  $\int V\rho dV$: This represents the total electric charge enclosed within the volume V bounded by the surface S.

  - Meaning: This form shows that the total outward electric flux from any closed surface is directly proportional to the total charge enclosed within it. This is a formal statement of Gauss's Law for electricity.
- Differential Form: $\nabla\cdot\vec{D}=\rho$
  - $\nabla\cdot\vec{D}$: This is the divergence of the electric displacement field. It measures the "net outward flow" of the electric field from an infinitesimal point.
  - ρ: This is the volume charge density at that point.
  - Meaning: This form describes the relationship at a single point in space. It states that if there is a net outward flow of the electric field at a point, there must be a positive charge density at that point. Conversely, a net inward flow indicates a negative charge density.

b) $R = (3,4,0) - (0,0,0) = 3a_x + 4a_y + 0a_z = 3a_x + 4a_y$

$|R| = \sqrt{(3^2+4^2)} = 5$

$F = Q_1\, Q_2\,.\, R / 4\pi\varepsilon_0 R^3 = 2x10^{-6}\ x\ {-5}x10^{-6}\ x\ (3a_x + 4a_y) / 4\pi\ x\ 10^{-9}/ 36\pi\ x\ 5^3$

$= (54a_x + 72a_y)$ mN

4

Calculate distances and potential V_p at P(-1,1,2)

The position vectors from the charges to point P are:

$\vec{R}_1 = P - \vec{r}_1 = (-1,1,2) - (0,0,4) = (-1,1,-2)$

$\vec{R}_2 = P - \vec{r}_2 = (-1,1,2) - (-2,5,1) = (1,-4,1)$

$\vec{R}_3 = P - \vec{r}_3 = (-1,1,2) - (3,-4,6) = (-4,5,-4)$

The magnitudes of these distances are:

$R_1 = |\vec{R}1| = \sqrt{[(-1)^2+1^2+(-2)^2} = \sqrt{6}$ m

$R_2 = |\vec{R}2| = \sqrt{[1^2+(-4)^2+1^2} = \sqrt{18}$ m

$R_3 = |\vec{R}_3| = \sqrt{[(-4)^2+5^2+(-4)^2} = \sqrt{57}$m

The potential V_p at P is the sum of potentials from each charge: $V_p = (Q1/R1 + Q2/R2 + Q_3/R_3)$

$V_p = 8.988\times10^9\ (1\times10^{-3}\sqrt{6} + -2\times10^{-3}\ \sqrt{18} + 3\times10^{-3}\ \sqrt{57})$

$V_p \approx 3.008\times10^6$ V

Calculate electric field E at P(-1,1,2)

The electric field $\vec{E}$ at P is the vector sum of electric fields from each charge:

$\vec{E} = (Q_1/R_1{}^3\ \vec{R}_1 + Q_2/R_2{}^3\ \vec{R}_2 + Q_3/R_{33}\ \vec{R}_3$

$\vec{E} = 8.988\times10^9\ (10^{-3}/(\sqrt{6})^3\ (-1,1,-2) + -2\times10^{-3}/(\sqrt{18})^3\ (1,-4,1) + 3\times10^{-3}/(\sqrt{57})^3\ (-4,5,-4)$

Calculating the components and summing:

$\vec{E} \approx (-1.11\times10^6\hat{\mathbf{i}} + 1.25\times10^6\hat{\mathbf{j}} - 1.95\times10^6\mathbf{k})$ V/m

Calculate potential difference V_{pq}

The potential difference $V_{pq} = V_p - V_q$. The potential V_q at (1,2,3) is calculated similarly:

Distances to Q:

$R_{1Q} = |Q - \vec{r}_1| = |(1,2,3) - (0,0,4)| = |(1,2,-1)| = \sqrt{[1^2+2^2+(-1)^2]} = \sqrt{6}$ m

$R_{2Q} = |Q - \vec{r}_2| = |(1,2,3) - (-2,5,1)| = |(3,-3,2)| = \sqrt{[3^2+(-3)^2+2^2]} = \sqrt{22}$ m

$R_{3Q} = |Q - \vec{r}_3| = |(1,2,3) - (3,-4,6)| = |(-2,6,-3)| = \sqrt{[(-2)^2+6^2+(-3)^2]} = 7$m

$V_q = (Q_1/R_{1Q} + Q_2/R_{2Q} + Q_3/R_{3Q})$

$V_q = 8.988\times10^9\ (1\times10^{-3}/\sqrt{6} + -2\times10^{-3}/\sqrt{22} + 3\times10^{-3}/7)$

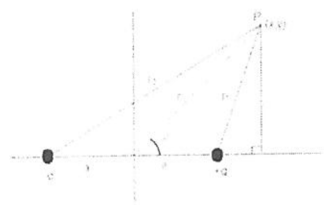
$V_q \approx 3.693\times10^6$ V

The potential difference is

$V_{pq} = V_p - V_q \approx 3.008\times10^6\ \text{V} - 3.693\times10^6\text{V} \approx -6.85\times10^5$ V.

5

a)



To derive the expression for the electric potential due to an electric dipole at an off-axis point, we start with the definition of electric potential due to a dipole. The electric potential V at a point in space due to a dipole is given by the formula:

$V = (1/\ 4\pi\epsilon0)\ \text{p}\cdot\text{r}/r^3$

where p is the dipole moment and r is the position vector from the dipole to the point of interest. For an off-axis point, we can express the dipole moment as p=pn^, where p is the magnitude of the dipole moment and n^ is the unit vector in the direction of the dipole. The distance r is the distance from the midpoint of the dipole to the point of interest. The angle θ between the dipole moment and the position vector can be used to express the dot product: p·r=$pr\cos(\theta)$. Substituting this into the potential formula gives us:

V=(1/4πϵ0). prcos(θ)/ r^3 = pcos(θ) /4πϵ0r^2

This is the expression for the electric potential due to an electric dipole at an off-axis point at a distance r from the midpoint of the dipole.

Step by Step Solution:

Step 1

Start with the formula for electric potential due to a dipole: V=(1/ 4$\pi\epsilon$0) p·r /r^3

Step 2

Define the dipole moment as p=pn^, where p is the dipole moment and n^ is the unit vector.

Step 3

Express the dot product p·r as p·r=$pr\cos(\theta)$, where θ is the angle between p and r.

Step 4

Substitute the dot product into the potential formula: V=(1/4$\pi\epsilon$0) $pr\cos(\theta)$ /r^3

Step 5

Simplify the expression to obtain: V= $p\cos(\theta)$ //4$\pi\epsilon$0r^2

Final Answer:

The expression for electric potential due to an electric dipole at an off-axis point at a distance r from the midpoint of the dipole is:

V=$p\cos(\theta)$ /4$\pi\epsilon$0r^2

b)

An electric dipole is a system of two equal and opposite charges separated by a small distance. The dipole moment ($\vec{p}$) is a vector quantity that quantifies the strength and orientation of this separation. Its magnitude is the product of the charge magnitude (q) and the distance (d) between the charges, and its direction points from the negative charge to the positive charge.

Electric Dipole

Definition: A pair of equal and opposite point charges, such as $+q$ and $-q$, that are separated by a specific distance.

Example: A water molecule (H_2O) is an electric dipole because the oxygen atom has a partial negative charge, while the hydrogen atoms have partial positive charges due to unequal sharing of electrons.

Dipole Moment ($\vec{p}$)

Definition: A vector quantity representing the strength and direction of the dipole.

Magnitude: Calculated as the product of the magnitude of one of the charges and the distance between the two charges: p=qd

If the distance between the charges is represented as $2a$ (where a is the distance from the center to each charge), the formula is p=($2a$).

Direction: The vector points from the negative charge to the positive charge.

SI Unit: Coulomb-meter ($C{\cdot}m$).

6

a)

The Biot-Savart law states that the magnetic field (dB) produced by a small segment of a current-carrying wire (Idl) is directly proportional to the current, the length of the segment, the sine of the angle (θ) between the segment and the line to the point, and inversely proportional to the square of the distance (r) from the segment to the point. It is used to calculate the magnetic field strength at a point from a current source, and its formula is $dB = (\mu_0/4\pi)\,.\,Idl\sin\theta/r^2$

Current and length (Idl): The magnetic field strength is directly proportional to the current and the length of the small current element (dl).
Angle ($\sin\theta$): The field is strongest when the current element is perpendicular to the line connecting it to the point ($\sin 90° = 1$) and weakest when it is parallel ($\sin 0° = 0$).
Distance (r^2): The magnetic field decreases with the square of the distance from the current element to the point of measurement, a relationship similar to Coulomb's law for electric fields.
Constant ($\mu 04\pi$): This is a constant of proportionality that depends on the magnetic properties of the medium. μ_0 is the permeability of free space.
Vector nature: The law is a vector equation. The direction of the magnetic field (dB) is perpendicular to both the current element (dl) and the position vector (r), and can be found using the right-hand rule.
Vector formula

The law is often expressed in vector form as: $d\vec{B} = (\mu_0/4\pi)\, Id\vec{l} \times \hat{r}/r^2$

$d\vec{l}$ is the vector for the current element, pointing in the direction of the current.
$\hat{r}$ is a unit vector pointing from the current element to the observation point.
The cross product ($d\vec{l} \times \hat{r}$) accounts for the direction of the magnetic field.

b)

Derivation using Ampère's Law

Set up the Amperian loop: Choose a rectangular Amperian loop with length $2L$ parallel to the current sheet and width W perpendicular to the sheet. Place the loop so that its sides of length $2L$ are at distances d above and below the sheet, and its sides of length W are perpendicular to the sheet.

Apply Ampère's Law: Ampère's Law states that the integral of H around a closed loop is equal to the total current enclosed by the loop, $Ienc$.

$$\oint \mathbf{H}\cdot d\mathbf{l} = Ienc$$

Evaluate the integral:

- The integral along the two sides perpendicular to the sheet is zero because the magnetic field is parallel to the current sheet, so $H\cdot dl = 0$.
- The integral along the two sides parallel to the current sheet is $2\cdot(H\cdot 2L)$, assuming H is constant and parallel to the loop on both sides.
- Therefore, the integral is $2HL$.

Determine the enclosed current: The total current enclosed by the loop is the current density (K) multiplied by the area it crosses ($2LW$).

$$Ienc = K\cdot(2L)$$

Solve for H: Equate the two sides of Ampère's Law and solve for the magnitude of the magnetic field intensity, H

$2HL = K\cdot 2L$

$H = K/2$

Determine the direction: The direction of the magnetic field is found using the right-hand rule. For a current flowing in the positive y direction, the magnetic field is in the $+x$ direction for points above the sheet ($z>0$) and in the $-x$ direction for points below the sheet ($z<0$).
Final expression: The magnetic field intensity can be expressed in vector form as:

- For $z>0$, $H=K/2\ \hat{ax}$
- For $z<0$, $H=-K/2\ \hat{ax}$
- The expression can also be written using the unit vector normal to the sheet, $\hat{a}_N$, as $H=K/2$ $(\hat{a}N\times\hat{k})$, where $\hat{k}$ is the unit vector in the direction of current.

7

a)

The capacitance (C) of a parallel plate capacitor is given by the formula $C=k\epsilon 0A/d$, where k is the dielectric constant, ϵ_0 is the permittivity of free space, A is the area of the plates, and d is the separation between the plates. The energy stored (U) in the capacitor has three equivalent expressions: $U=1/2CV^2$, $U=1/2QV$, and $U=1/2Q^2C$.

Capacitance

- Formula: $C=k\epsilon_0A/d$

  Variables:

  C: Capacitance (in Farads, F)
  k: Dielectric constant (1 for a vacuum)
  ϵ_0: Permittivity of free space ($\approx 8.854\times 10^{-12}$ F/m)
  A: Area of one plate (in m^2)
  d: Distance between the plates (in meters, m)

Energy Stored

- Formula 1: $U=1/2CV^2$
- Formula 2: $U=1/2QV$
- Formula 3: $U=1/2Q^2C$
- Variables:

  U: Energy stored (in Joules, J)
  C: Capacitance (in Farads, F)
  V: Voltage across the capacitor (in Volts, V)
  Q: Charge on one plate (in Coulombs, C)

b)

The magnetic field intensity **H** at a point P due to a differential current element $Id\mathbf{L}$ in free space is

$$d\mathbf{H} = Id\mathbf{L}\times\mathbf{R}\ /4\pi R^3$$

where **R** is the vector from the current element to the observation point, and R is its magnitude.

$$\mathbf{H}=\mathbf{H}_1+\mathbf{H}_2$$

Calculate $\mathbf{H}_1$

Current element 1: $\mathbf{I}_1 d\mathbf{L}_1=4\times 10^{-5}\ \mathbf{a}_x$ A·m at $P_1(0,0,0)$.
Observation point: $(3,1,-2)$.
Vector **R**1 from $P1$ to P: $\mathbf{R}_1 = P-P1 = (3\mathbf{a}x+1\mathbf{a}y-2\mathbf{a}z) - (0,0,0) = 3\mathbf{a}x+1\mathbf{a}y-2\mathbf{a}z$
Magnitude R_1: $R_1 = \sqrt{[3^2+1^2+(-2)^2]} = \sqrt{(9+1+4)} = \sqrt{14}$
Cross product $\mathbf{I}_1 d\mathbf{L}_1\times\mathbf{R}_1$: $\mathbf{I}_1 d\mathbf{L}_1\times\mathbf{R}_1 = (4\times 10^{-5}\mathbf{a}_x) \times (3\mathbf{a}_x+1\mathbf{a}_y-2\mathbf{a}_z)$
$=4\times 10^{-5}(1(\mathbf{a}_x\times\mathbf{a}_y)-2(\mathbf{a}_x\times\mathbf{a}_z)) = 4\times 10^{-5}(1\mathbf{a}_z-2(-\mathbf{a}_y))$
$=4\times 10^{-5}(2\mathbf{a}_y+\mathbf{a}_z)=(8\times 10^{-5})\mathbf{a}_y+(4\times 10^{-5})\mathbf{a}_z$
Field $\mathbf{H}_1$: $\mathbf{H}1=(8\times 10^{-5})\mathbf{a}_y+(4\times 10^{-5})\mathbf{a}_z\ /4\pi(14\sqrt{})3\approx 1.22\times 10^{-7}\mathbf{a}_y+0.61\times 10^{-7}\mathbf{a}_z$ A/m

Calculate $\mathbf{H}_2$

Current element 2: $\mathbf{I}_2 d\mathbf{L}_2 = 6\times10^{-5}\mathbf{a}_y$A·m at $P_2(0,0,1)$.

Vector $\mathbf{R}_2$ from P_2 to P: $\mathbf{R}_2 = P-P_2 = (3\mathbf{a}_x+1\mathbf{a}_y-2\mathbf{a}_z)-(0,0,1) = 3\mathbf{a}x+1\mathbf{a}y-3\mathbf{a}z$

Magnitude R_2: $R_2 = \sqrt{[3^2+1^2+(-3)^2} = \sqrt{(9+1+9)} = \sqrt{19}$

Cross product $\mathbf{I}_2 d\mathbf{L}_2\times\mathbf{R}_2$: $\mathbf{I}_2 d\mathbf{L}_2\times\mathbf{R}_2 = (6\times10^{-5}\mathbf{a}_y)\times(3\mathbf{a}_x+1\mathbf{a}_y-3\mathbf{a}_z)$

$=6\times10^{-5}(3(\mathbf{a}_y\times\mathbf{a}_x)-3(\mathbf{a}_y\times\mathbf{a}_z))=6\times10^{-5}(3(-\mathbf{a}_z)-3(\mathbf{a}_x))$

$=6\times10^{-5}(-3\mathbf{a}_x-3\mathbf{a}_z) = (-18\times10^{-5})\mathbf{a}_x-(18\times10^{-5})\mathbf{a}_z$

Field $\mathbf{H}_2$:

$$H2=(-18\times10^{-5})\mathbf{a}_x-(18\times10^{-5})\mathbf{a}_z/4\pi(\sqrt{19})^3 \approx -1.73\times10^{-7}\mathbf{a}_x-1.73\times10^{-7}\mathbf{a}_z\text{A/m}$$

Calculate total H

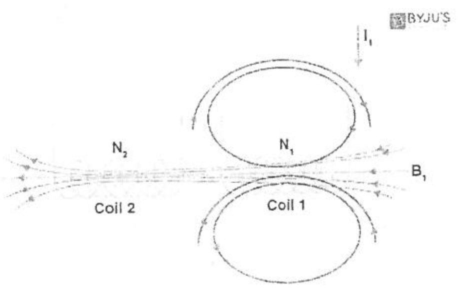
$$\mathbf{H}=\mathbf{H}_1+\mathbf{H}_2$$

$$\mathbf{H} \approx (-1.73\times10^{-7})+(1.22\times10^{-7})\mathbf{a}_y+(0.61\times10^{-7}-1.73\times10^{-7})\mathbf{a}_z$$

$$\mathbf{H}\approx-1.73\times10^{-7}\mathbf{a}_x+1.22\times10^{-7}\mathbf{a}_y-1.12\times10^{-7}\mathbf{a}_z\text{A/m}$$

8

a)



The self-inductance (L) of a solenoid is calculated using the formula $L=\mu 0N^2A/l$, where μ_0 is the permeability of free space, N is the total number of turns, A is the cross-sectional area, and l is the length of the solenoid. If the solenoid has a core made of a magnetic material, the μ_0 is replaced with the material's magnetic permeability ($\mu=\mu_r\mu_0$), where μ_r is the relative permeability of the core.

Formula

For an air-core solenoid: $L=\mu_0 N^2A/l$

For a solenoid with a magnetic core: $L=\mu N^2A/l$ or $L=\mu_r\mu_0 N^2A/l$

Components of the formula

L: Self-inductance (measured in Henries)

μ_0: Permeability of free space ($\approx4\pi\times10^{-7}$ T·m/A)

μ: Magnetic permeability of the core material

μ_r: Relative permeability of the core material (for air, $\mu_r=1$)

N: Total number of turns of wire

A: Cross-sectional area of the solenoid (in m^2)

l: Length of the solenoid (in meters)

How to calculate

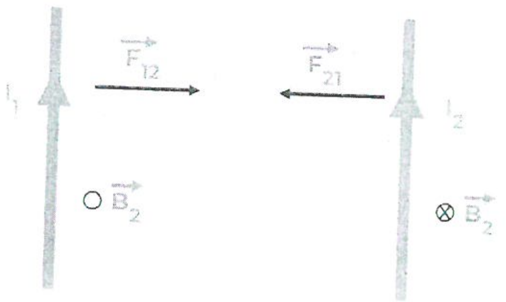
Measure the physical properties: Determine the number of turns (N), the cross-sectional area (A), and the length (l) of the solenoid.

Identify the core material: Determine if the solenoid has a core and find its relative permeability (μ_r). If it's an air-core solenoid, use $\mu_r=1$.

Apply the formula:

Plug the values into the appropriate formula to find the self-inductance. For a solenoid with a core, you can also use the formula $L = (\mu_0 N^2A/l)\times\mu_r$

b)



Two parallel, current-carrying conductors exert a magnetic force on each other, which is attractive if the currents are in the same direction and repulsive if they are in opposite directions. The magnitude of this force is directly proportional to the product of the currents (I_1 and I_2), the length of the conductors (L), and inversely proportional to the distance (r) between them, as described by the formula: $F=\mu_0 I_1 I_2 L/2\pi r$.

Nature of the force

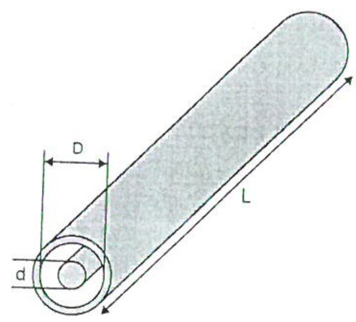
- Attractive: When the currents in both conductors flow in the same direction, the conductors pull towards each other.
- Repulsive: When the currents in the conductors flow in opposite directions, they push away from each other.

Magnitude of the force

The magnitude of the force per unit length ($f=F/L$) between two long, parallel conductors is given by the formula: $f=\mu_0 I_1 I_2 /2\pi r$.
I_1 and I_2 are the currents in the two conductors. r is the distance between the conductors. μ_0 is the permeability of free space.

9

a)



Find the magnetic field (B) using Ampere's Law:

- For a coaxial cable with a current I in the inner conductor and $-I$ in the outer conductor, the magnetic field in the region between the conductors ($a<r<b$) is given by Ampere's Law.
- Consider an Amperian loop of radius r concentric with the conductors. The enclosed current is I.
- The magnetic field is tangential and has a constant magnitude along the loop: $B\cdot(2\pi r)=\mu_0 I$.
- Therefore, the magnetic field at a distance r is $B=\mu 0 I/2\pi r$.

Calculate the magnetic flux (Φ) through a section of the cable:

- The flux through a small rectangular strip of length l and width dr at a distance r is $d\Phi = B \cdot dA$.
- The area of this strip is $dA = l \cdot dr$.
- So, $d\Phi = (\mu_0 I/2\pi r) \cdot (l \cdot dr) = (\mu_0 I l/2\pi r) dr$.
- To find the total flux, integrate this expression from the inner radius (a) to the outer radius (b): $\Phi = \int_a^b d\Phi = \int_a^{b}(\mu_0 I l/2\pi r) dr = \mu_0 I l/2\pi \int_a^b 1/r \, dr$.
- The integral of $1/r$ is $\ln(r)$:
$$\Phi = (\mu_0 I l/2\pi)[\ln(r)]^b{}_a = (\mu_0 I l/2\pi)(\ln(b) - \ln(a)) = (\mu_0 I l/2\pi)\ln(b/a)$$

Solve for the self-inductance (L):

- The definition of self-inductance is $L = \Phi/I$.
- Substitute the expression for Φ: $L = (1/I)(\mu_0 I l/2\pi)\ln(b/a)$.

The current I cancels out, leaving: $L = (\mu_0 l/2\pi)\ln(b/a)$

b)

The expression for the force on a current element in a magnetic field is derived by considering the Lorentz force on each charge carrier and summing them over the length of the conductor, leading to the vector formula $\vec{F} = I(\vec{L} \times \vec{B})$. For a straight conductor, this simplifies to the scalar magnitude $F = ILB\sin\theta$, where I is the current, L is the length of the conductor, B is the magnetic field strength, and θ is the angle between the current and the magnetic field.

Derivation steps

Start with the Lorentz force on a single charge: The fundamental magnetic force on a single charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by the Lorentz force law:

$\vec{F}e = q(\vec{v} \times \vec{B})$.

Consider the drift velocity of charge carriers: In a current-carrying conductor, the charge carriers (e.g., electrons) have a drift velocity, $\vec{v}_d$. The force on a single charge is therefore

$\vec{F}e = (\vec{v}d \times \vec{B})$.

Relate drift velocity to current: The current (I) is the rate of flow of charge. For a conductor with a cross-sectional area A and charge carrier density n, the current is $I = nA_q v_d$. This can be rewritten to relate charge q to current I: $q = I/nv_d A$.

Consider a small length element: Consider a small segment of the conductor of length ΔL. The volume of this segment is $A\Delta L$. The number of charge carriers in this segment is $N = nA\Delta L$.

Sum the forces: The total force on this small segment is the sum of the forces on each charge carrier. This is given by: $\vec{F} = N\vec{F}e = (nA\Delta L)q(\vec{v}_d \times \vec{B})$.

Group the terms: Rearrange the terms: $\vec{F} = (nqAvd)(\Delta L \times \vec{B})$.

Substitute current and length vector: Substitute the expression for current, $I = nqAv_d$: $\vec{F} = (\Delta\vec{L} \times \vec{B})$, where $\Delta\vec{L}$ is a vector in the direction of the current with magnitude ΔL.

Integrate for a continuous current: For a continuous current, the total force is found by integrating the force on each element along the entire length of the conductor: $\vec{F} = (\vec{L} \times \vec{B})dl$. If the magnetic field is uniform and the conductor is straight, this simplifies to $\vec{F} = (\vec{L} \times \vec{B})$, where $\vec{L}$ is the vector representing the length of the conductor.

Magnitude of the force

The magnitude of this vector force is given by the scalar equation: $F = ILB\sin\theta$

where:

F is the magnitude of the force

I is the current flowing through the conductor

L is the length of the conductor in the magnetic field

B is the magnitude of the magnetic field

θ is the angle between the direction of the current ($\vec{L}$) and the magnetic field ($\vec{B}$)

10

a)

The expression for a statically induced EMF in a time-varying field is given by Faraday's Law as

$E = -d\Phi_B/dt$,

where,

E is the induced electromotive force and

$d\Phi_B/dt$ is the rate of change of magnetic flux (Φ_B) over time. This can also be expressed as

$E = -N d\Phi/dt$,

where

N is the number of turns in the coil and

Φ is the magnetic flux through each turn.

b)

Static fields remain constant over time and are produced by stationary sources (like stationary charges), while time-varying fields change in magnitude and/or direction over time, generated by accelerated charges or time-varying currents

. A key difference is that time-varying fields induce other fields (e.g., a changing magnetic field induces an electric field), a phenomenon not possible with static fields.

Feature	Static Fields	Time-Varying Fields
Definition	Fields with constant magnitude and direction over time.	Fields that change in magnitude and/or direction over time.
Sources	Stationary electric charges (for electric fields) and steady currents (for magnetic fields).	Accelerated charges or time-varying currents (for both electric and magnetic fields).
Interaction	Do not induce new fields.	A changing magnetic field can induce an electric field, and a changing electric field can induce a magnetic field.
Mathematical Representation	Static electric fields are described by equations where time derivatives are zero.	Time-varying fields are represented as functions of both space and time (x,y,z,tx comma y comma z comma t x,y,z,t).
Curl	Static electric fields have a curl of zero (non-rotational).	Time-varying fields have a non-zero curl due to the time-derivative terms

11

a)

Stationary Loop in time varying B Field

As integration over space and differentiation with time are independent operations,

$-d/dt \int_s B.dS = - \int_s d/dt\,(B.dS)$

V_{emf} = closed$\int_L$ E.dl = - $\int_s$ (δB / δt) .dS

Apply Stoke's theorem,

$\int_s$ (∇ x E) dS = - $\int_s$ (δB / δt) .dS

(∇ x E) = - (δB / δt)

b)

When electromagnetic waves travel in space, they transport energy. Energy density is linked to the electric and magnetic fields.

The Poynting vector represents the rate of energy propagation per unit area perpendicular to the incident energy flow.

$P=E\times H P=\boldsymbol{E}\times\boldsymbol{H}$

$P=EHsin\quad\theta P=EHsin\theta$

The electric field (E) and magnetic field (H) vectors are oriented perpendicular to each other and to the direction of wave propagation.

where P is rate of energy transfer and poynting vector

$P=E\times B\mu P=\mu\boldsymbol{E}\times\boldsymbol{B}$

Proof of Poynting Theorem

Statement: At any point in a uniform plane wave, the cross product of electric field E and magnetic field H gives a measure of the rate of energy flow per unit area at that point.

$P=E\times H P=\boldsymbol{E}\times\boldsymbol{H}$

using the Maxwell equation We can Write as:

∇×H=J+∂D∂t∇×**H**=**J**+∂*t*∂**D** -(1)

∇×E=−∂B∂t∇×**E**=−∂*t*∂**B** -(2)

taking a dot product with E

E·(∇×H)=E·J+E·∂D∂t**E**·(∇×**H**)=**E**·**J**+**E**·∂*t*∂**D** -(3)

using identity, we can Write

∇·(E×H)=H·(∇×E)−E·(∇×H)∇·(**E**×**H**)=**H**·(∇×**E**)−**E**·(∇×**H**)

or

E·(∇×H)=H·(∇×E)−∇·(E×H)**E**·(∇×**H**)=**H**·(∇×**E**)−∇·(**E**×**H**)

put the value of Equation 3

H·(∇×E)−∇·(E×H)=E·J+E·∂D∂t**H**·(∇×**E**)−∇·(**E**×**H**)=**E**·**J**+**E**·∂*t*∂**D** -(4)

∇·(E×H)=H·(∇×E)−E·J−E·∂D∂t∇·(**E**×**H**)=**H**·(∇×**E**)−**E**·**J**−**E**·∂*t*∂**D** -(5)

From Equation 2, take dot product with H

∇·(∇×E)=−(H·∂H∂t)κ∇·(∇×**E**)=−(**H**·∂*t*∂**H**)*κ*

....where ϰϰ = permeability

put the value in Equation 5

∇·(E·H)=−(H·∂H∂t)κ−E·J−(E·∂E∂t)ϵ∇·(**E**·**H**)=−(**H**·∂*t*∂**H**)*κ*−**E**·**J**−(**E**·∂*t*∂**E**)*ϵ*-(6) Where D=ϵE*D*=*ϵE*

Lets us consider the term

∂∂t(H·H)=H·∂H∂t+H·∂H∂t=2(H·∂H∂t)=12∂∂t(H2)∂*t*∂(**H**·**H**)=**H**·∂*t*∂**H**+**H**·∂*t*∂**H**=2(**H**·∂*t*∂**H**)=21∂*t*∂(**H**2)

Similarly, we can Write as

E·∂E∂t=12∂∂t(E2)**E**·∂*t*∂**E**=21∂*t*∂(**E**2)

put these values in Equation 6

$\nabla\cdot(E\times H)=-\kappa 2\partial H2\partial t-E\cdot J-\epsilon 2\partial E2\partial t\nabla\cdot(\mathbf{E}\times\mathbf{H})=-2\kappa\partial t\partial\mathbf{H}2-\mathbf{E}\cdot\mathbf{J}-2\epsilon\partial t\partial\mathbf{E}2$

Taking volume integral

$\int v\nabla\cdot(E\times H)dv=-\mu 2\int v\partial\partial t(H22)dv-\int v(E\cdot J)dv-\epsilon 2\int v\partial\partial t(E2)dv\int v\nabla\cdot(\mathbf{E}\times\mathbf{H})dv=-2\mu\int v\partial t\partial(2\mathbf{H}2)dv-\int v(\mathbf{E}\cdot\mathbf{J})dv-2\epsilon\int v\partial t\partial(\mathbf{E}2)dv$

$\oint\nabla\cdot(E\times H)dv=-\int v(E\cdot J)dv-\partial\partial t\int v(\mu H2/2+\epsilon E2/2)dv\oint\nabla\cdot(\mathbf{E}\times\mathbf{H})dv=-\int v(\mathbf{E}\cdot\mathbf{J})dv-\partial t\partial\int v(\mu\mathbf{H}2/2+\epsilon\mathbf{E}2/2)dv$ (rate of energy flow) and $\int v(E\cdot J)dv\int v(\mathbf{E}\cdot\mathbf{J})dv=$ power loss, $\partial\partial t\int v(\epsilon E22+\mu H22)dv\partial t\partial\int v(2\epsilon\mathbf{E}2+2\mu\mathbf{H}2)dv=$rate of decrease in electromagnetic wave energy.

Hence, poynting theorem is proved

In a uniform plane wave, the first term and next term combine, then something energy loss occurs. These energy rates per unit area are called the poynting vector.

$P=E\times HP=\mathbf{E}\times\mathbf{H}$