PVP 23

Code: 23BS1302

II B.Tech - I Semester – Regular / Supplementary Examinations NOVEMBER 2025

NUMERICAL METHODS AND COMPLEX VARIABLES
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Prove that $E\nabla = \Delta = \nabla E$	L2	CO1
1.b)	Evaluate $\Delta(e^{2x+3})$	L2	CO1
1.c)	Write the formula of first derivative of the function $y = f(x)$ at $x = x_n$ using Newton's backward Interpolation formula.	L3	CO2
1.d)	State Eulers's method formula.	L2	CO1
1.e)	Write the Cauchy – Riemann (C-R) equations in cartesian form.	L2	CO1
1.f)	Prove that the function $u(x,y) = 3xy^2 - x^3$ is harmonic.	L3	CO3
1.g)	Expand ze^z as Maclaurin's series.	L3	CO3
1.h)	Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$	L2	CO1

1.i)	Write the zero's and the poles of $f(z)=\dfrac{z^4+z^3}{z^3(1-z)}$	L3	CO3
1.j)	Determine the residue of the function $\frac{9z+1}{z(z^2+1)}$ at z=0	L3	CO3

PART – B

<table>
<tr><td></td><td></td><td></td><td>BL</td><td>CO</td><td>Max. Marks</td></tr>
<tr><td colspan="6">UNIT-I</td></tr>
<tr><td rowspan="2">2</td><td>a)</td><td>Find the missing values in the following table
<table>
<tr><td>x</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr>
<tr><td>y</td><td>6</td><td>10</td><td>--</td><td>17</td><td>--</td><td>31</td></tr>
</table></td><td>L4</td><td>CO4</td><td>5 M</td></tr>
<tr><td>b)</td><td>Apply bisection method to find a real root of the equation $x^3-4x-9=0$.</td><td>L3</td><td>CO2</td><td>5 M</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td rowspan="2">3</td><td>a)</td><td>Estimate the population in the year 1895 using Newton's forward interpolation formula from the following statistics:
<table>
<tr><td>Year x:</td><td>1891</td><td>1901</td><td>1911</td><td>1921</td><td>1931</td></tr>
<tr><td>Population y:</td><td>46</td><td>66</td><td>81</td><td>93</td><td>101</td></tr>
</table></td><td>L4</td><td>CO4</td><td>5 M</td></tr>
<tr><td>b)</td><td>Estimate the value of y at x=10 using Lagrange's interpolation formula from the following table
<table>
<tr><td>x</td><td>5</td><td>6</td><td>9</td><td>11</td></tr>
<tr><td>y</td><td>12</td><td>13</td><td>14</td><td>16</td></tr>
</table></td><td>L4</td><td>CO4</td><td>5 M</td></tr>
<tr><td colspan="6"></td></tr>
</table>

<table>
<tr><td colspan="6">UNIT-II</td></tr>
<tr><td>4</td><td>a)</td><td>Using forward difference formulae, obtain $\frac{dy}{dx}$ at $x = 2$ for the data given below:
<table><tr><td>x:</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>y:</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr></table></td><td>L3</td><td>CO2</td><td>5 M</td></tr>
<tr><td></td><td>b)</td><td>Evaluate $\int_0^1 x^3$ dx with five subintervals taking h=0.2 by trapezoidal rule.</td><td>L4</td><td>CO4</td><td>5 M</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>5</td><td colspan="2">Estimate $y(0.2)$ in steps of 0.1 using Runge-Kutta 4th order (R-K) method, given that
$\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}, \; y(0) = 1.$</td><td>L4</td><td>CO4</td><td>10 M</td></tr>
<tr><td colspan="6">UNIT-III</td></tr>
<tr><td>6</td><td>a)</td><td>Prove that
$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |\text{Real } f(z)|^2 = 2|f'(z)|^2$
where $w = f(z) = u + iv$ is analytic.</td><td>L4</td><td>CO5</td><td>5 M</td></tr>
<tr><td></td><td>b)</td><td>Show that $v(x, y) = -\sin x \sinh y$ is harmonic. Find the conjugate harmonic of $v(x, y)$ where $w = f(z) = u + iv$ is analytic.</td><td>L3</td><td>CO3</td><td>5 M</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>7</td><td colspan="2">Construct an analytic function $f(z) = u + iv$ whose real part is $u = e^x(x\, sin\, y + y\, cos\, y)$</td><td>L3</td><td>CO3</td><td>10 M</td></tr>
<tr><td colspan="6">UNIT-IV</td></tr>
<tr><td>8</td><td>a)</td><td>Evaluate $\oint_C z^2\, dz$ around the square with the vertices at (0,0), (1,0),(1,1), (0,1)</td><td>L4</td><td>CO5</td><td>5 M</td></tr>
</table>

	b)	Using Cauchy's integral formula, evaluate $\oint_C \frac{e^z}{z(1-z)^3} dz$, where C is $\lvert z\rvert = \frac{1}{2}$	L4	CO5	5 M
OR					
9	a)	Expand the Laurent series of $f(z) = \frac{1}{z^2-3z+2}$ in the region $0 < \lvert z-1\rvert < 1$	L3	CO3	5 M
	b)	Find the Taylor's series expansion of the function $\frac{2z^3+1}{z^2+z}$ about the point z=1.	L3	CO3	5 M
UNIT-V					
10	a)	Determine and classify the singularities of $\frac{z}{e^z-1}$	L3	CO3	5 M
	b)	Evaluate $\int_C \frac{5z-2}{z(z-1)} dz$ where C is $\lvert z\rvert = 2$	L3	CO3	5 M
OR					
11		Evaluate $\int_0^{2\pi} \frac{d\theta}{3+2\cos\theta}$ using contour integration in the complex plane.	L4	CO5	10 M