Unit-5
Graph Theory

A graph G is a Pair of sets (V, E) where V is a set of vertices & E is a set of edges. If G is a directed graph (digraph) the elements of E are odered pair of vertices.

A graph with no loops is said to be simple or loop-free.

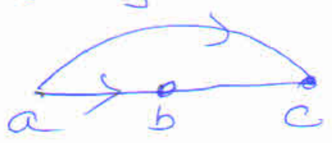
Directed Path :- A directed Path in a graph $G(V, E)$ is a sequence of edges $e_1, e_2, e_3, \ldots e_n$ in E such that $e_i = (v_{i-1}, v_i)$ where $i = 1, 2, \ldots n$.

(A graph G is a Pair of sets (V, E) where V is a non-empty set & the elements of E are pair of elements of V)

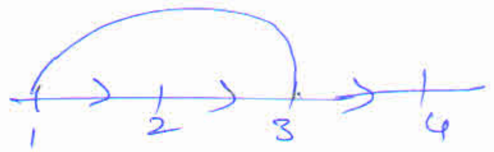
If the elements of E are ordered Pairs denoted as (u, v) then G is called a directed graph (diagraph)

If the elements of E are unordered Pairs denoted as $\{u, v\}$ then G is called a non-directed graph.

Example $G = \{V, E\}$.
where $V = \{a, b, c\}$ & $E = \{(a,b)\ (b,c)\ (a,c)\}$ is a directed graph.



Example :



Directed Path $= \{(1,2)\ (2,3)\ (3,4)\}$. $V = \{1,2,3,4\}$.
No Directed Path. vertices 1,4 are end Points of the Path.

c **Length of the Path** :- The length of the Path is defined as the no. of edges involved in the Path.

Example. The length of the Path is 3. (1,2)(2,3)(3,4)

End Points of the Path :- A directed Path in a graph $G=(V,E)$ is a sequence of edges $e_1, e_2, \ldots e_n$ in a E such that $e_i = (v_{i-1}, v_i)$ $i = 1,2,\ldots n$ here v_0, v_n are called End Points of the Path.

E

non-directed Path :- A non directed Path in a diagraph $G=(V,E)$ is a sequence of zero or more edges $e_1, e_2, \ldots e_n$ in E for which there is a sequence of vertices $v_0, v_1 \ldots v_n$ such that $e_i = (v_{i-1}, v_i)$ or $e_i = (v_i, v_{i-1})$ for each $1 < i \leq n$.

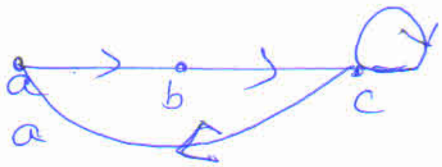
A Path is **Simple** if all edges & vertices on the path are ~~donot~~ distinct Except the End Points may be Equal.

A **Path of length ≥ 1** with no repeated edges & whose End Points are Equal is a **circuit**

A simple circuit is called a circle.

cycle :- A simple circuit is

Example



(a,b) (b,c) (c,a) circle

(a,b) (b,c) (c,c) (c,a) circuit

(a,b) (b,c) simple path

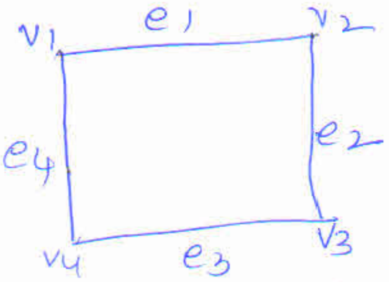
Note 1. Every circuit is need not be a Simple Path
2. Every Simple Path need not be a cycle.

In a graph $G=(V,E)$ an edges which is associated with an ordered Pairs of $V \times V$ is Called a directed edge of G, while an edge which is associated with an un ordered Pair of nodes is Called an undirected edge. A graph in which every edge is directed is Called a diagraph or directed graph. A graph in which every edge is undirected is Called an undirected graph

Unit - V

Graphs

Graphs:- Vertices & edges:- A Graph G is a Pair of sets (V, E) where V is a set of vertices and E is a set of edges.



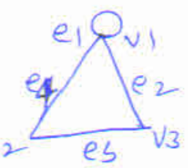
$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$.

$e_1 = \{v_1, v_2\}$ $e_2 = \{v_2, v_3\}$ $e_3 = \{v_3, v_4\}$

$e_4 = \{v_4, v_1\}$ $e_5 = \{v_2, v_4\}$

loop An edge drawn from a vertex to itself is called loop.



$V = \{v_1, v_2, v_3\}$ $E = \{e_1, e_2, e_3, e_4\}$ $e_1 = \{v_1, v_1\}$

The edge e_1 is called a loop.

Simple graph or loop free:- A graph with no loops is said to be simple graph or loop free.

Parallel edges:- In a graph, if some pair of vertices are Joined by more than one edge, such edges are called Parallel edges.

$V = \{v_1, v_2, v_3\}$ $E = \{e_1, e_2, e_3, e_4\}$

$e_1 = \{v_2, v_1\}$ $e_2 = \{v_1, v_3\}$ $e_3 = \{v_3, v_1\}$

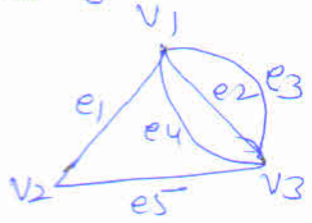
e_2 & e_3 are Parallel edges.

Simple graph:- A graph without loops & Parallel edges is called simple graph.

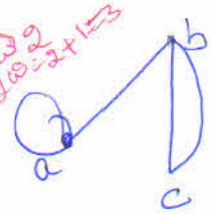
Order & size: If a graph $G = (V, E)$ is a finite graph, then the number of vertices in G is called order of a graph G. It is denoted by $|V(G)|$ or $|V|$. number of edges in a graph G is called size of a graph. It is denoted by $|E(G)|$ or $|E|$.

If one allows more than one edge to join a pair of vertices then it is called Multigraph.

Degree:- The degree of a vertex is determined by counting each loop incident on a vertex "b" as twice and each other edge incident on vertex b as once. It is denoted by deg(v).



loopకి d = 2
a's loopకి 1 = 2+1=3

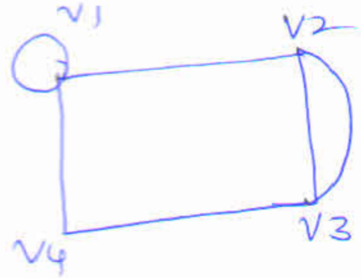


$d(a) = 3$
$d(b) = 3$
$d(c) = 2$.

order of the graph $|V| = 3$
size of the graph $|E| = 4$.

Problem

1. Find the degree of all the vertices



$d(v_1) = 2+1+1 = 4$
$d(v_2) = 1+1+1 = 3$
$d(v_3) = 3$ $\quad d(v_4) = 2$.

The minimum degree of a graph G is denoted as $\delta(G)$ and defined as the minimum degree of all the vertices of a graph G.

The Maximum degree of a graph G is denoted as $\Delta(G)$.

$\delta(G) = \min$ of $\deg(v_i)$, $v_i \in V$

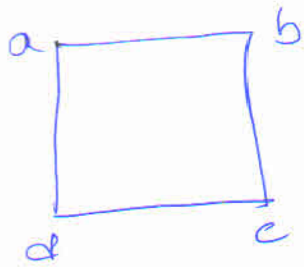
$= \{4, 3, 3, 2\} = 2.$

$\Delta(G) = \text{Max}$ of $\deg(v_i)$ $v_i \in V$

$= \{4, 3, 3, 2\} = 4.$

Definition k-Regular graph

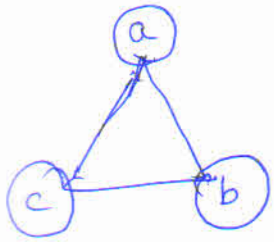
A graph G is said to be a k-Regular graph if degree of each vertex is k. i.e $\delta(G) = \Delta(G) = k.$



$d(a) = 2$
$d(b) = 2$
$d(c) = 2$
$d(d) = 2$

$\Delta G = \delta(G) = 2$

$\therefore$ 2-Regular graph.



$d(a) = 4$
$d(b) = 4$
$d(c) = 4.$

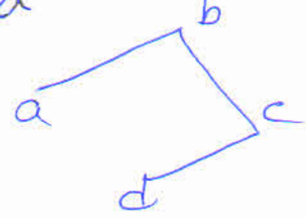
$\delta(G) = \Delta(G) = 4.$

4-Regular graph.

For any graph G we have $\delta(G) \leq \deg(v_i) \leq \Delta(G)$. $\forall$ $v_i \in V$.

Let G be a graph with vertex $V = \{v_1, v_2, \ldots v_n\}$ and degree $d_i = \deg(v_i)$ then the sequence $d_1, d_2 \ldots d_n$ is called the degree sequence of a graph G.

Graphic:- A sequence $D = \{D_1, D_2 \ldots D_n\}$ is graphic if there exits a simple graph with degree sequence d



$d(a) = 1$, $d(b) = 2$, $d(c) = 2$, $d(d) = 1$

degree sequence $\{1, 2, 2, 1\}$

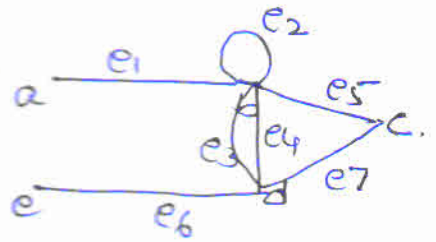
Note A walk in graph is a finite alternating sequence of vertices and edges i.e. $v_0 e_1 v_1 e_2 v_2 \ldots e_n v_n$ such that $e_i = (v_{i-1}, v_i)$, $1 \le i \le n$.

Note $v_0 = v_n$ it is called a closed walk otherwise it is an open walk.

Example:- $a e_1 b e_2 b e_4 d$ is an open walk.

$a e_1 b e_2 b e_1 a$ is a closed walk.

$a e_1 b e_6 c$ — not a walk.



Note:- A walk is called a trail if all its edges are distinct.

Note:- An open trail is called a path and a closed trail is called a circuit.

Note:- A circuit in which all the vertices are distinct except the first & last vertices is called a cycle.

$b e_5 c e_7 d$ — Path $b e_5 c e_7 d e_4 b$ — circuit

$b e_4 d e_3 b e_4 d$ — walk but not trail

$b e_5 c e_7 d e_4 b$ — closed walk $\Rightarrow$ circuit

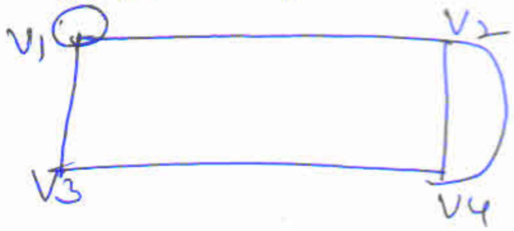
$b e_5 c e_7 d e_4 b e_2 b$ — not a cycle but a circuit since b is repeated.

Note:- A path of length ≥ 1 with no repeated edges and whose end points are equal is called a circuit.

Note:- A circuit may have repeated vertices other than end points.

Note:- $\delta(G)$ is denoted as the minimum of all the degrees of the vertices of a graph G and $\Delta(G)$ is denoted as the Maximum of all the degrees of the vertices of G.

Example Find $\delta(G)$, $\Delta(G)$ for the graph.



Vertex	Degree
v_1	4
v_2	3
v_3	2
v_4	3

$\delta(G) =$ Minimum of $\{\text{degree}(v_i) \mid v_i \in V\}$

$=$ Minimum of $\{4, 3, 2, 3\}$

$= 2.$

$\Delta(G) =$ Maximum of $\{\text{degree}(v_i) \mid v_i \in V\}$

$=$ Maximum of $\{4, 3, 2, 3\}$

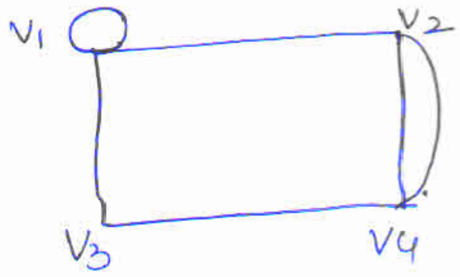
$= 4.$

Note:- For any graph G we have $\delta(G) \leq \deg(v_i) \leq \Delta(G)$ for all $v_i \in V$.

Definition: Degree of Sequence: Let G be a graph with vertex set $V = \{v_1, v_2, \ldots v_n\}$ and let $d_i = \deg(v_i)$ then the sequence $(d_1, d_2 \ldots d_n)$ is called the degree sequence of G.

The vertices of a graph G can be ordered so that the degree sequence is monotonically increasing or monotonically decreasing.

Problem: Find the degree sequence of the graph



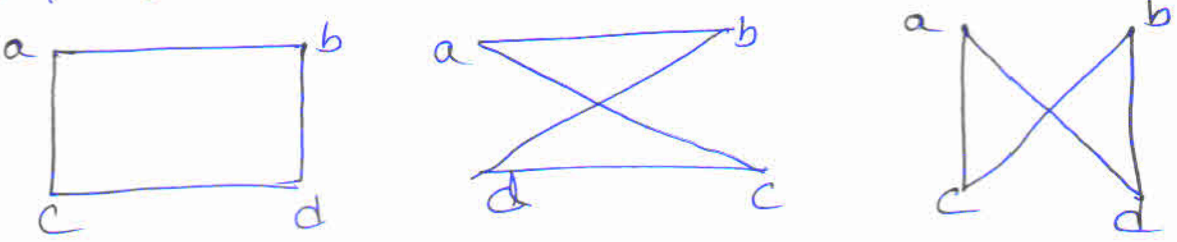
Sol. The degree sequence for the vertex ordering v_1, v_2, v_3, v_4 is $(4, 3, 2, 3)$

We can write the degree sequence in increasing order as $(2, 3, 3, 4)$. The vertex ordering for this is v_3, v_2, v_4, v_1.

K-Regular :- A graph G is said to be K-regular if each vertex in it is of degree K. That is $\delta(G) = \Delta(G) = K$.

Example :- Draw all 2-regular graphs of order 4.

(i)



Theorem: (The sum of degrees theorem)

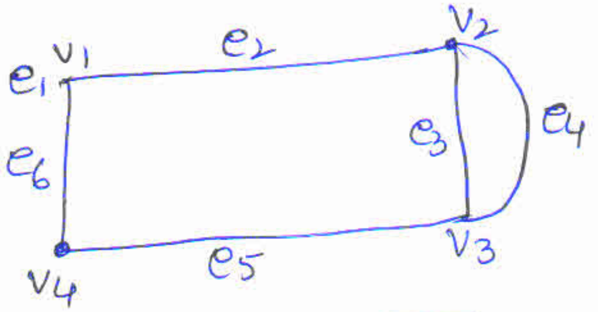
Let G be a graph with $|V| = n$ vertices and $|E|$ edges

then $\sum_{i=1}^{n} \deg(v_i) = 2|E|$.

Proof: Let $v_1, v_2, \ldots v_n$ be the vertices of a graph G.
Since each edge has two end points and each end contributes one to the sum of degrees then the sum of degrees of all the vertices in G is twice the number of edges in G.

i.e $\sum_{i=1}^{n} \deg(v_i) = 2|E|$.

Example:- Verify the sum of degrees theorem to the following graph.



Sol:-

Vertex	v_1	v_2	v_3	v_4
Degree	4	3	3	2

$\sum \deg(v_i) = 4+3+3+2 = 12$.

$2(E) = 2 \times$ number of edges
$= 2 \times 6$
$= 12$.

$\sum \deg(v_i) = 2|E|$.

Problem :- Find the number of edges in a graph containing 2 vertices of degree 3, 3 vertices of degree 4 and a vertex of degree six.

Sol :- By sum of degrees theorem.

$$\sum \deg(v_i) = 2|E|.$$

$$2(3) + 3(4) + 1(6) = 2(E)$$

$$6 + 12 + 6 = 2(E)$$

$$24 = 2(E)$$

$$12 = |E|.$$

$\therefore$ The number of Edge in G is 12

Problem : Find the number of vertices in a graph containing 3 vertices of degree 4, 2 vertices of degree 3 and remaining vertices of degree 2. Given that number of edges in G is 11.

Sol : By sum of degree's theorem

$$\sum \deg(v_i) = 2|E|$$

$$3(4) + 2(3) + x(2) = 11(2)$$

$$12 + 6 + 2x = 22$$

$$18 + 2x = 22$$

$$x = 2.$$

$\therefore$ The number of vertices in G is $3 + 2 + x$

$= 3 + 2 + 2 \quad [\because x = 2]$

$= 7.$

Note :- If any graph, the number of odd degree vertices is an Even number.

Note If G is a graph with $|V|$ vertices and $|E|$ edges, then $\delta(G) \leq \frac{2|E|}{|V|} \leq \Delta(G)$.

Note If G is a k-regular graph then

$$k \cdot |V| = 2|E|$$

Problem :- Find the number of edges in a 4-regular graph of order 3.

Sol :- Given G is 4-regular

Here $k = 4$, $|V| = 3$.

We know that $k|V| = 2|E|$

$$4 \times (3) = 2 \times |E|$$

$$12 = 2|E|$$

$$|E| = 6$$

The number of edges in G is 6.

Problem :- Is there a graph with degree sequence $\{1, 3, 3, 3, 4, 4, 5, 6\}$?

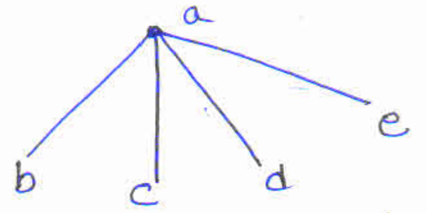
Sol: No, since the number of vertices with odd degree is not even number as we have 5 odd degrees 1, 3, 3, 3, 5.

Graphic :- A Sequence $d = (d_1, d_2, \ldots d_n)$ is graphic if there Exists a simple graph with degree Sequence d.

Problem :- Is the degree Sequence (2, 2, 3, 3, 4) a graphic

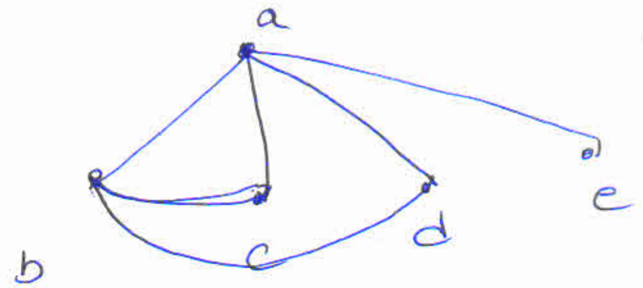
Sol :- we should have 5 vertices. Let these be a, b, c, d, e with degrees respectively 4, 3, 3, 2, 2.

We first draw all 4 edges from a to other vertices b, c, d, e

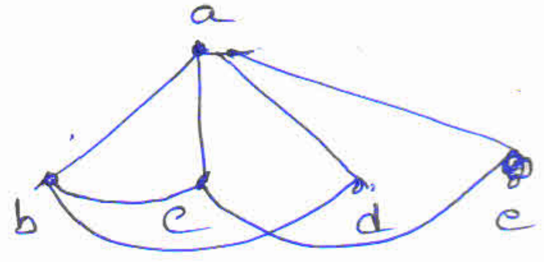


∴ The number of edges adjacent to the vertices b, c, d, e in the remaining Portion of the graph respectively 2, 2, 1, 1 (b c d e)

Now draw 2 edges from the vertex b.



In the remaining Portion of the graph the number of edges adjacent to c, d, e are respectively 1, 0, 1.



Thus the degree Sequence (2, 2, 3, 3, 4) is graphic.

Note:- the degree sequence $(d_1, d_2 \cdots d_n)$ is graphic iff $(d_2-1, d_3-1, d_4-1, \cdots d_{d_i}-1, d_{d_i+1}-1, d_{d_i+2} \cdots + d_n)$ is graphic.

(Assume $d_1 \geq d_2 \geq \cdots \geq d_n$)

Problem:- Is there a simple graph with degree sequence

(1, 3, 3, 3, 4, 5, 5)

Sol. we first write the sequence in descending order

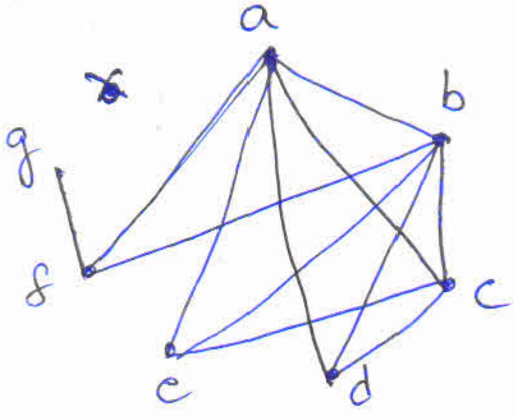
5, 5, 4, 3, 3, 3, 1

Delete the first 5 and substract one from each of the next 5 terms, we get the sequence 4, 3, 2, 2, 2, 1

Now delete the first 4 and substract one from each of the next 4 terms we get the sequence 2, 1, 1, 1, 1

Now we delete the first 2 and substract one from each of the next terms given 0, 0, 1, 1 which is graph

a	b	c	d	e	f	g
~~5~~	5	4	3	3	3	1
	~~4~~	3	2	2	2	1
		~~2~~	1	1	1	1
			0	0	1	1

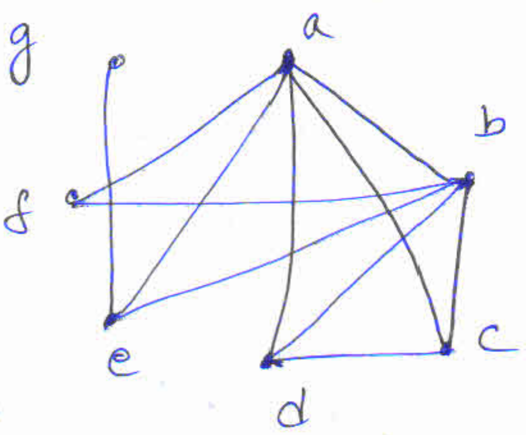


Problem :- Is the degree Sequence (1, 2, 3, 3, 3, 5, 5) a graphic ?

Sol :- we first write the sequence in descending order

5, 5, 3, 3, 3, 2, 1.

a	b	c	d	e	f	g
~~5~~	5	3	3	3	2	1
	~~4~~	2	2	2	1	1
		~~1~~	1	1	0	1
			0	1	0	1



a → 5 lines.
b → 4 → lines
c → 1
e to g → line.

which is graphic. The given degree sequence is graphic.

Problem :- Is there a simple graph with degree sequence (1, 1, 3, 3, 3, 4, 6, 7) ?

Sol we first write the Sequence in descending order

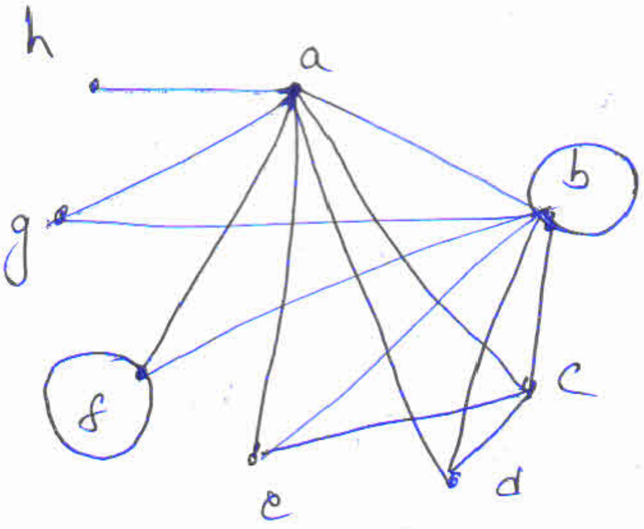
a	b	c	d	e	f	g	h
~~7~~	6	4	3	3	3	1	1
	~~5~~	3	2	2	2	0	1
		2	1	1	1	-1	

which is not graphic. since we cannot find a vertex with degree 5. [simple graph means no loops and no parallel edges].

Problem :- Is there a graph with degree sequence $(1, 1, 3, 3, 3, 4, 6, 7)$?

Sol:

a	b	c	d	e	f	g	h	
$\cancel{7}$	6	4	3	3	3	1	1	
	$\cancel{5}$	3	2	2	2	0	0	
	$\cancel{3}$	3	2	2	2	0	0	(draw a loop at b)
		$\cancel{2}$	1	1	2	0	0	
			0	0	2	0	0	(draw a loop at f)

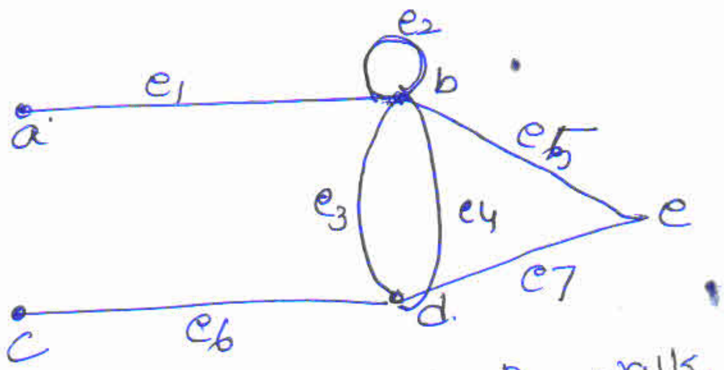


This is a graph.

Definition (walk) :- A walk in a graph is a finite alternating sequence of vertices and edges $v_0 e_1 v_1 e_2 v_2 \cdots e_n v_n$ such that $e_i = (v_{i-1}, v_i)$, $1 \le i \le n$.

closed walk and open walk :- If $v_0 = v_n$ it is called a closed walk, otherwise it is an open walk.

Example :- (*)



$a\, e_1\, b\, e_2\, b\, e_4\, d$ is an open walk. length = 3.
$a\, e_1\, b\, e_2\, b\, e_1\, a$ is a closed walk. length = 3.
$a\, e_1\, b\, e_6\, c$ is not a walk.

Trail :- A walk is called a trail if all its edges are distinct.
Path :- An open trail is called a Path.
circuit :- A closed trail is called a circuit.
cycle :- A Circuit in which all the vertices are distinct Except the first and last vertices is called a cycle.

Example :- In Previous Example (*).

$b\, e_5\, e\, e_7\, d$ is a Path.

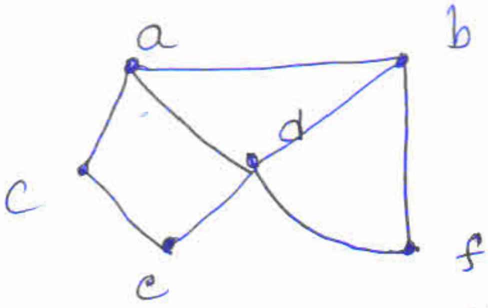
$b\, e_5\, e\, e_7\, d\, e_4\, b$ is a circuit

$b\, e_4\, d\, e_3\, b\, e_4\, d$ is a walk but not a trail.

$b\, e_5\, e\, e_7\, d\, e_4\, b$ is a cycle.

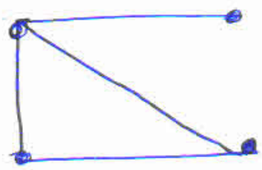
$b\, e_5\, e\, e_7\, d\, e_4\, b\, e_2\, b$ is not a cycle but a circuit since b is repeated.

Problem:- Given an Example of a circuit which is not a cycle in a Simple graph.



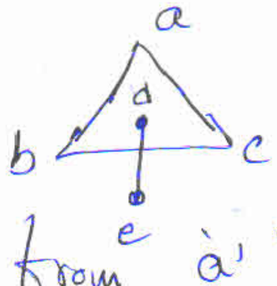
$a-d-f-b-d-e-c-a$ is a circuit but not a cycle since an internal vertex d is repeated.

Connected :- A graph is called Connected if there exists a path from a vertex to any other vertex.

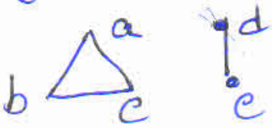


is Connected graph.

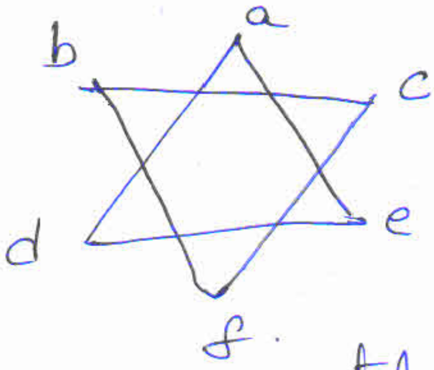
Example :- The graph



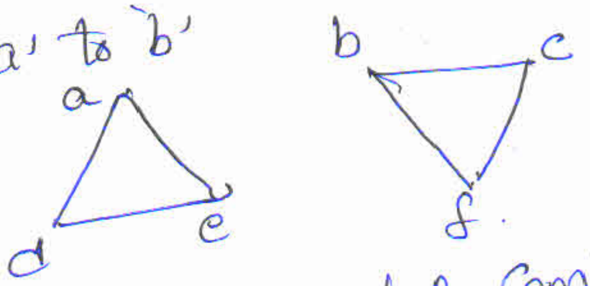
is not Connected since there is no Path from "a" to "d". It has two Connected Components



Example :- write the Connected Components of the following graph.



The graph is not Connected Since there is no path from 'a' to 'b'



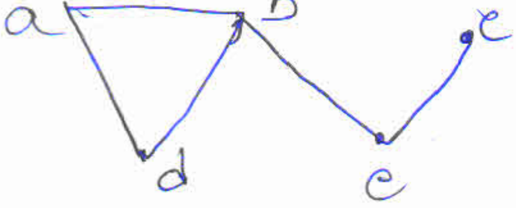
are the two Connected Components.

AdJacency Matrix :- The adJacency Matrix of a simple graph $G = (V, E)$ where $V = \{v_1, v_2, \dots v_n\}$ are the ordered vertices is an $n \times n$ Matrix

$$A_G = (a_{iJ})$$

$$\text{where } a_{iJ} = \begin{cases} 1 & \text{if an edge connects } v_i \text{ to } v_J \\ 0 & \text{if no edge } \text{ " } \quad \text{ " } \quad \text{ " } \quad \text{ " } \end{cases}$$

Problem :- Determine the adJacency Matrix for the simple graph shown in the following figure.



we first order the vertices as a, b, c, d, e.

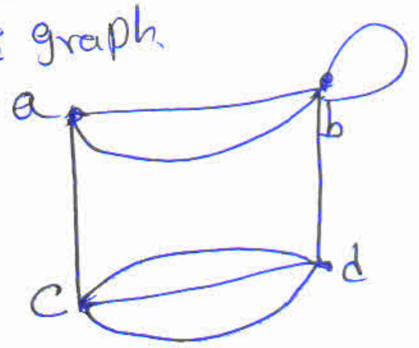
The adjacency matrix of the graph is

	a	b	c	d	e
a	0	1	0	1	0
b	1	0	0	1	1
c	0	0	0	0	1
d	1	1	0	0	0
e	0	1	1	0	0

Note:- 1. If there is a loop at the vertex v_i we write $a_{ii} = 1$

2. If there is more than one edge between the vertices v_i and v_j we write a_{ij} = number of edges between v_i and v_j.

Problem:- Write the adjacency matrix of the following multi graph.



Let a, b, c, d be the ordered vertices of the multigraph G.

The adjacency matrix of G is

	a	b	c	d
a	0	2	1	0
b	2	1	0	1
c	1	0	0	3
d	0	1	3	0

<u>Note</u> :- A cycle is a circuit with no other repeated vertices except its end points.

<u>Note</u> :- A cycle is a simple circuit.

<u>Isomorphism</u> :- Two graphs G_1 and G_2 are isomorphic if there is a function $f: V(G_1) \rightarrow V(G_2)$ such that

(i) f is 1–1 (one-one)

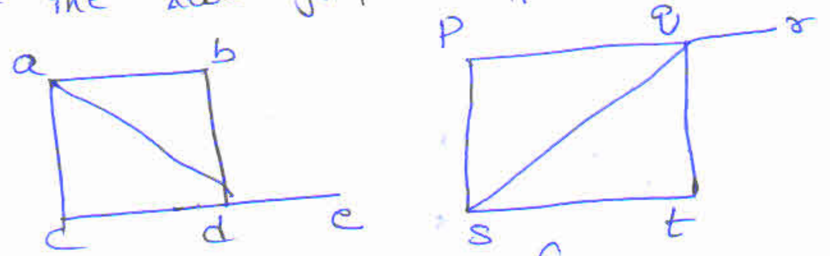
(ii) f is onto and

(iii) f preserves adjacency

i.e for each pair of vertices u and v of a graph G $\{u,v\} \in E(G_1)$ iff $\{f(u), f(v)\} \in E(G_2)$

<u>Problems</u> :-

1. S.T the two graphs G_1 and G_2 are isomorphic.



G_2

G_1

<u>Sol</u>: No. of vertices in $G_1 = |V_1| = 5$

No. of vertices in $G_2 = |V_2| = 5$

No. of Edges in $G_1 = |E_1| = 6$

No. of Edges in $G_2 = |E_2| = 6$

The two graphs G_1 and G_2 are of same order & size.

vertex	a	b	c	d	e
degree	3	2	2	4	1

G_1

vertex	p	q	r	s	t
degree	2	4	1	3	2

degree sequence $d_1 = \{4, 3, 2, 2, 1\}$

degree sequence $d_2 = \{4, 3, 2\,2, 1\}$

$F(d) = q \quad f(a) = s.$

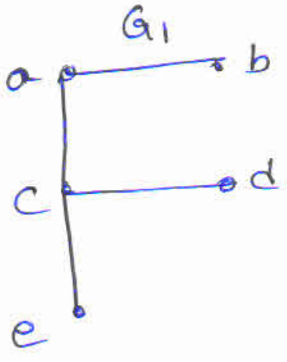
$F(b) = p$ or t ; $F(c) = t$ or p. ; $f(e) = r$.

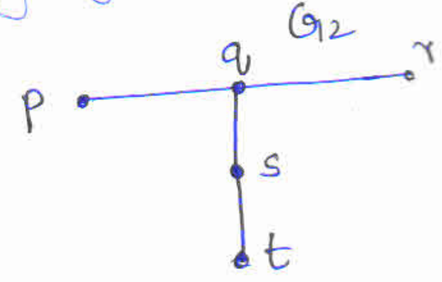
AG_1	a	b	c	d	e
a	0	1	1	1	0
b	1	0	0	1	0
c	1	0	0	1	0
d	1	1	1	0	1
e	0	0	0	1	0.

AG_2	s	p	t	q	r
s	0	1	1	1	0
p	1	0	0	1	0
t	1	0	0	1	0
q	1	1	1	0	1
r	0	0	0	1	0

Here $AG_1 = AG_2$ Since f is one-one, onto and preserves adjacency. The two graphs G_1 and G_2 are isomorphic.

Problem 2 :- S.T the following graphs are isomorphic.





Sol:- Number of vertices in Graph G_1 is Equal to the number of vertices in Graph G_2.

$|V_1|$ = Vertices $G_1 = \{a, b, c, d, e\} = 5$

$|V_2|$ = Vertices $G_2 = \{p, q, r, s, t\} = 5$

Number of edges in Graph G_1 is Equal to the number of edges in Graph G_2 i.e

$|E_1| = 4$., $|E_2| = 4$.

$$\left.\begin{array}{l}(a,b)=e_1\\(a,c)=e_2\\(c,d)=e_3\\(c,e)=e_4\end{array}\right\} = 4 = |E_1|. \qquad \left.\begin{array}{l}(p,q)=e_1\\(q,r)=e_2\\(q,s)=e_3\\(s,t)=e_4\end{array}\right\} = 4 = |E_2|$$

$\therefore$ The two graphs of G_1, G_2 are of ~~same~~ Same order & size

G_1

vertex	a	b	c	d	e
degree	2	1	3	1	1

G_2

vertex	p	q	r	s	t.
degre	1	3	1	2	1

Degree Sequence of d_1 (3, 2, 1, 1, 1)

Degree Sequence of d_2 (3, 2, 1, 1, 1)

We observe that both the degree sequence is also Same.

Let us derive $f: V_1 \to V_2$ as $f(c) = q$

$F(a) = s$

$F(b) = t$

$F(d) = p$ or $f(d) = r$

$F(e) = r$ or $f(e) = p$.

V	a	b	c	d	e
$f(V)$	s	t	q	r	p

AG_1	a	b	c	d	e
a	0	1	1	0	0
b	1	0	0	0	0
c	1	0	0	1	1
d	0	0	1	0	0
e	0	0	1	0	0

AG_2	s	t	q	r	p
s	0	1	1	0	0
t	1	0	0	0	0
q	1	0	0	1	1
r	0	0	1	0	0
p	0	0	1	0	0

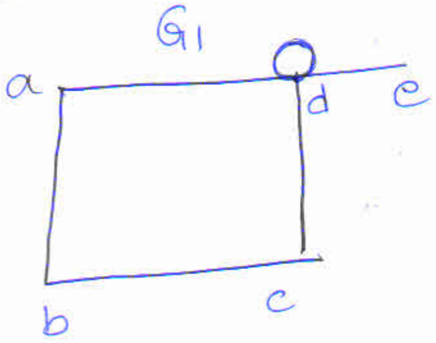
$\therefore$ $AG_1 = AG_2$.

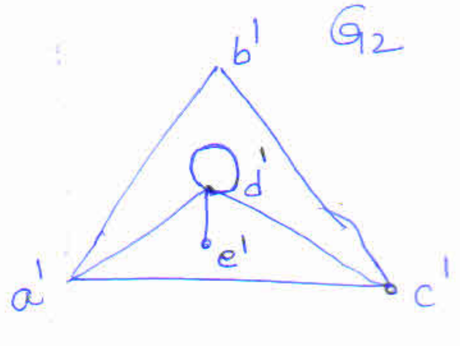
$\Rightarrow$ f Preserves adjacency.

Since f is one-one, onto & Preserves adjacency.

The Graph G_1 & G_2 are isomorphic.

Problem :- S.T The two graphs are isomorphic.





Sol:- (i) number of vertices in G_1 is same as number of vertices in G_2.

$|V_1| = \{a, b, c, d, e\} = 5$

$|V_2| = \{a', b', c', d', e'\} = 5$.

(ii) numbe of edges in G_1 is same as number of edges in G_2

$|E_1| = |E_2|$

The two Graphs G_1 & G_2 are same order & size.

G_1

vertex	a	b	c	d	e
degree	3	2	3	5	1

G_2

vertex	a'	b'	c'	d'	e'
degree	3	2	3	5	1

degree sequence of $d_1 = \{5, 3, 3, 2, 1\}$

degree sequence of $d_2 = \{5, 3, 3, 2, 1\}$

Let us define $f: V_1 \to V_2$.

$F(a) = a'$ or c'

$F(c) = c'$ or a'

$F(b) = b'$

$F(d) = d'$

$F(e) = e'$

(or) $f(a) = c'$

$F(c) = a'$

$F(b) = b'$

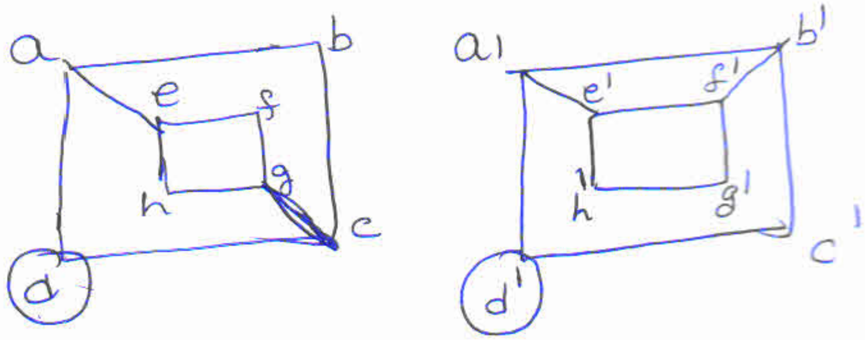
$F(d) = d'$

$F(e) = e'$

	a	b	c	d	e
a	0	1	1	1	0
b	1	0	1	0	0
c	1	1	0	1	0
d	1	0	1	1	1
e	0	0	0	1	0

	a'	b'	c'	d'	e'
a'	0	1	1	1	0
b'	1	0	1	0	0
c'	1	1	0	1	0
d'	1	0	1	1	1
e'	0	0	0	1	0

Problem :- S.T the two graphs are not isomorphic



$|V_1| = 8$ vertices $|V_2| = 8$.

Edges $|E_1| = 11$ Edges $|E_2| = 11$

The two graphs are of same order and size.

Vertex	a	b	c	d	e	f	g	h
degree	3	2	3	4	3	2	3	2

vertex	a'	b'	c'	d'	e'	f'	g'	h'
degree	3	3	2	4	3	3	2	2

In a graph G_1, It has two cycles of length 4

i.e (a-b-c-d-a) & e-f-g-h-e.

In a graph G_2 It has ~~two~~ three cycles of length 4.

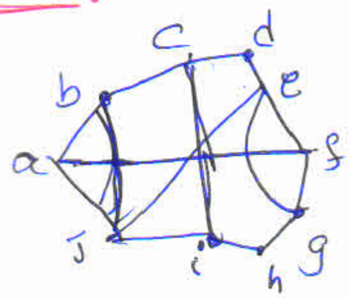
i.e a'-b'-c'-d'-a' & e'-f'-g'-h'-e' & a'-e'-f'-b'-a'.

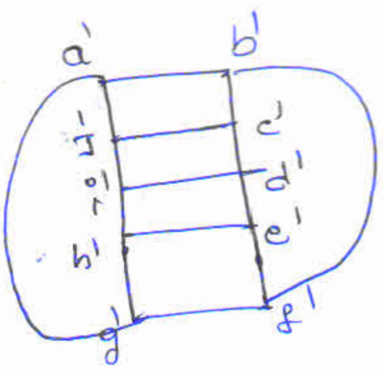
In the graph G_1, no pair of vertices of degree 2 are adjacent where as in the graph G_2, g' and h' are the vertices of degree 2 that are adjacent.

In the graph G_1, the vertex of degree 4 is adjacent to two vertices of degree 3 where as in the graph G_2, the vertex of degree 4 is adjacent to a vertex of degree 2 and 5,

$\therefore$ The two graphs G_1 and G_2 are not isomorphic.

Problem :- S.T the two graphs are not isomorphic.





Vertices $|V_1| = 10$,
Vertices $|V_2| = 10$
Edges $|E_1| = 15$
Eges $|E_2| = 15$.

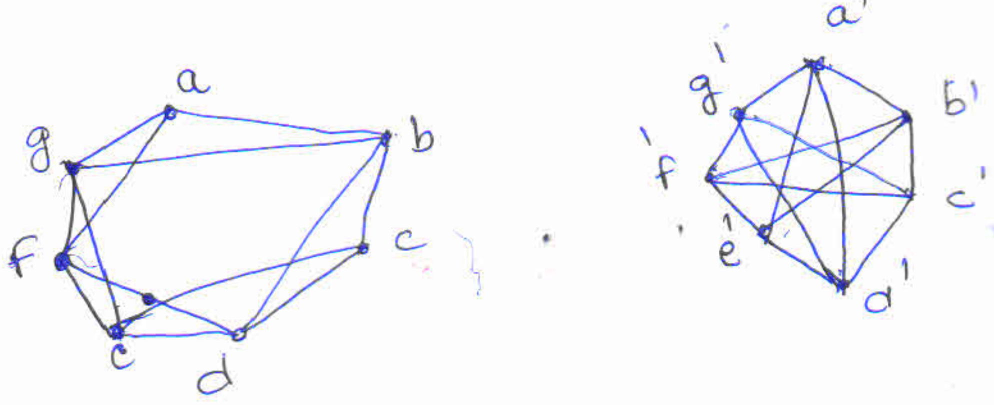
Vertex	a	b	c	d	e	f	g	h	i	j
degree	3	3	3	3	3	3	3	3	3	3

Vertex	a'	b'	c'	d'	e'	f'	g'	h'	i'	j'
degree	3	3	3	3	3	3	3	3	3	3

These two graphs have 10 vertices & 15 edges & all vertices of degree 3. In the graph G_1, It has two cycles of length 3 i.e a-b-J-a and e-f-g-e. Where as in the graph G_2. It has no cycle of length 3.

∴ The two graphs G_1 and G_2 are not isomorphic.

Problem The graphs are isomorphic.



$a \to a'$
$b \to e'$
$c \to b'$
$d \to f'$
$e - c'$
$f - g'$
$g - d'$